2-Hadron Interactions from Lattice QCD

4th Berkeley School on
Collective Dynamics in High-Eenergy Collisions
May 14- 18, 2012

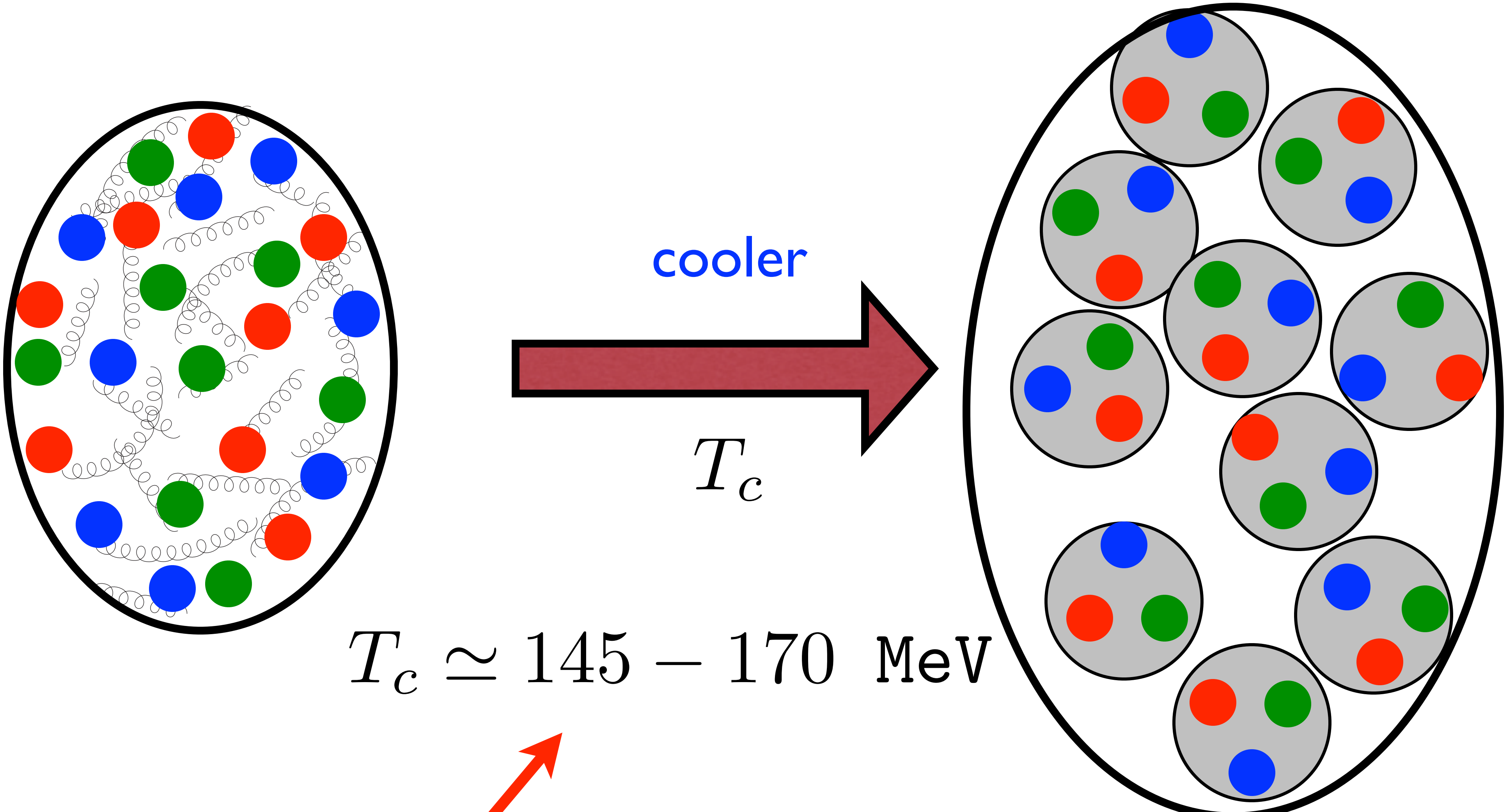
André Walker-Loud

- Importance of QCD in Nuclear Physics
  - Fine Tunings
- Scattering on the Lattice
- Challenges for Nuclear Physics on the Lattice
- Examples

Importance of QCD in Nuclear Physics

- Phase transition from QGP to Hadrons
- Big Bang Nucleosynthesis and primordial hydrogen and helium abundance
- Production of Carbon 12
- Equation of state of dense nuclear matter: neutron stars
- ...

Confinement of Quarks



$$T_c \simeq 145 - 170 \ \mathtt{MeV}$$

QCD: computed on BlueGene-L by my LLNL colleagues,
refined by Budapest-Wuppertal Collaboration

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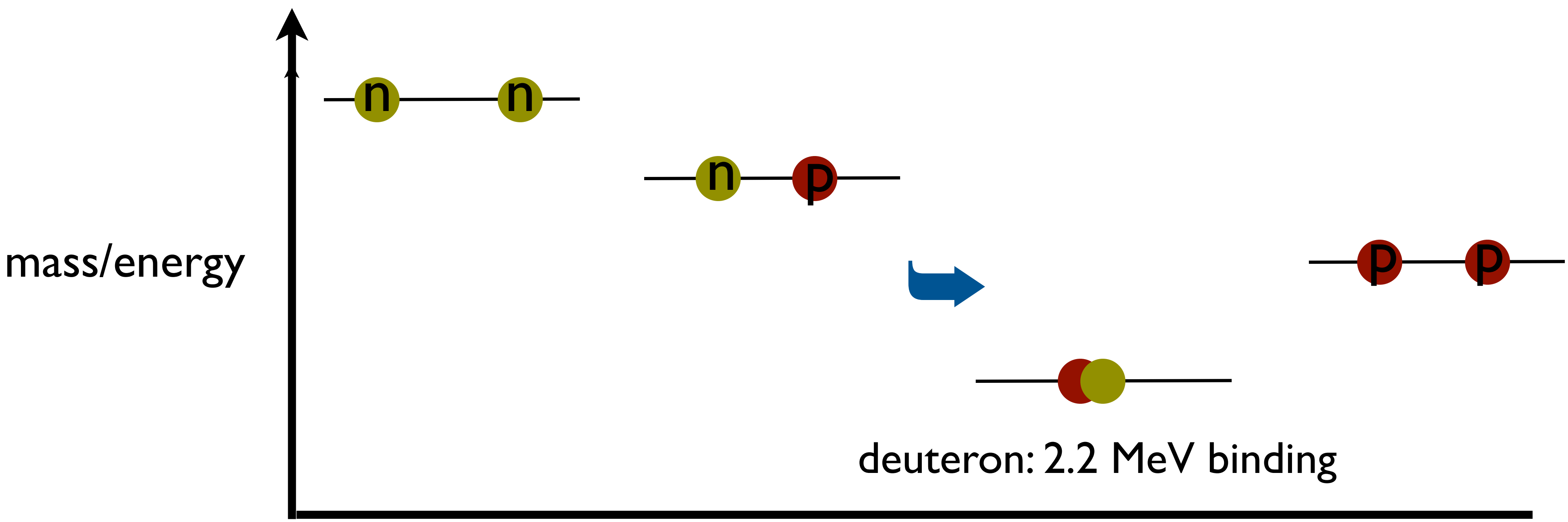
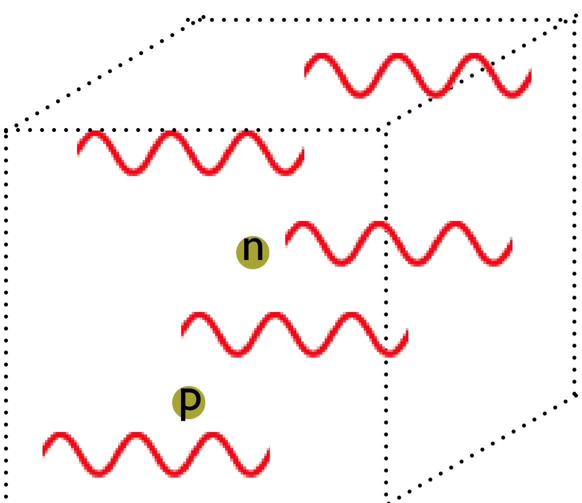
The deuterium "bottleneck"



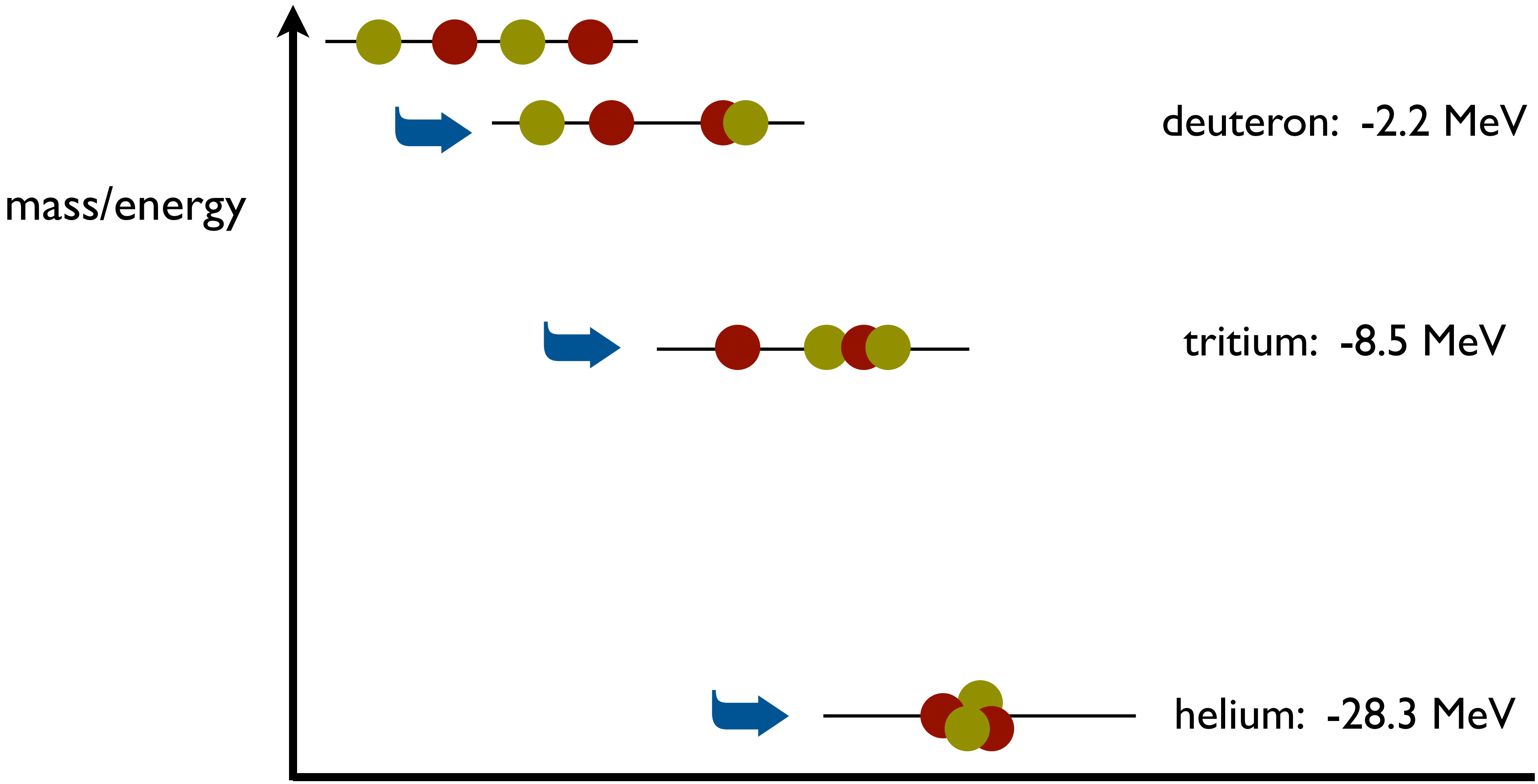


$$p + n \longleftrightarrow d + \gamma$$

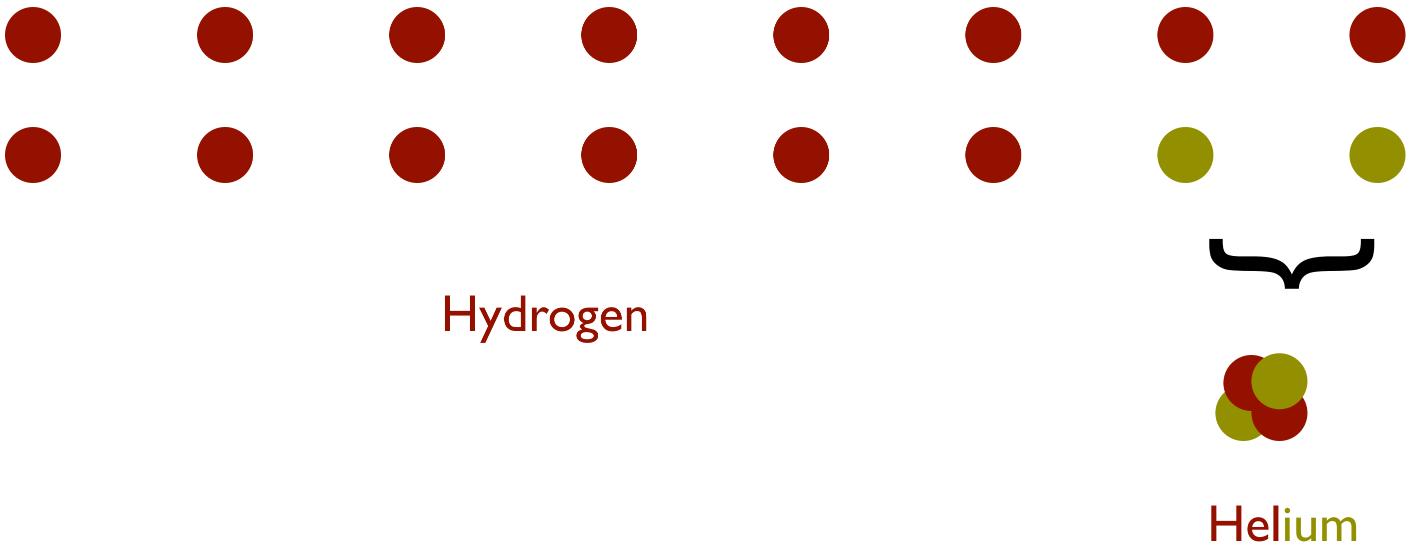
until T ≃ 100 keV (1 billion K), t ≃ 3 min

$$p + n \longrightarrow d + \gamma$$

The deuterium “bottleneck” is broken, neutrons flow into He



He stability: ↑,↓ protons and ↑,↓ neutrons can be packed together



The early universe contains 75% H and 25% He by mass fraction this picture very sensitive to binding energy of deuterium which is finely tuned (most nuclei have ~8 MeV binding per nucleon)!

$$B_d = 2.22 \ \texttt{MeV}$$

What if

$B_d \ll 2.22$ MeV more finely tuned

all neutrons decay - no helium

mostly hydrogen stars?

$B_d \gg 2.22$ MeV natural scenario

all neutrons captured in deuterium and helium - no hydrogen

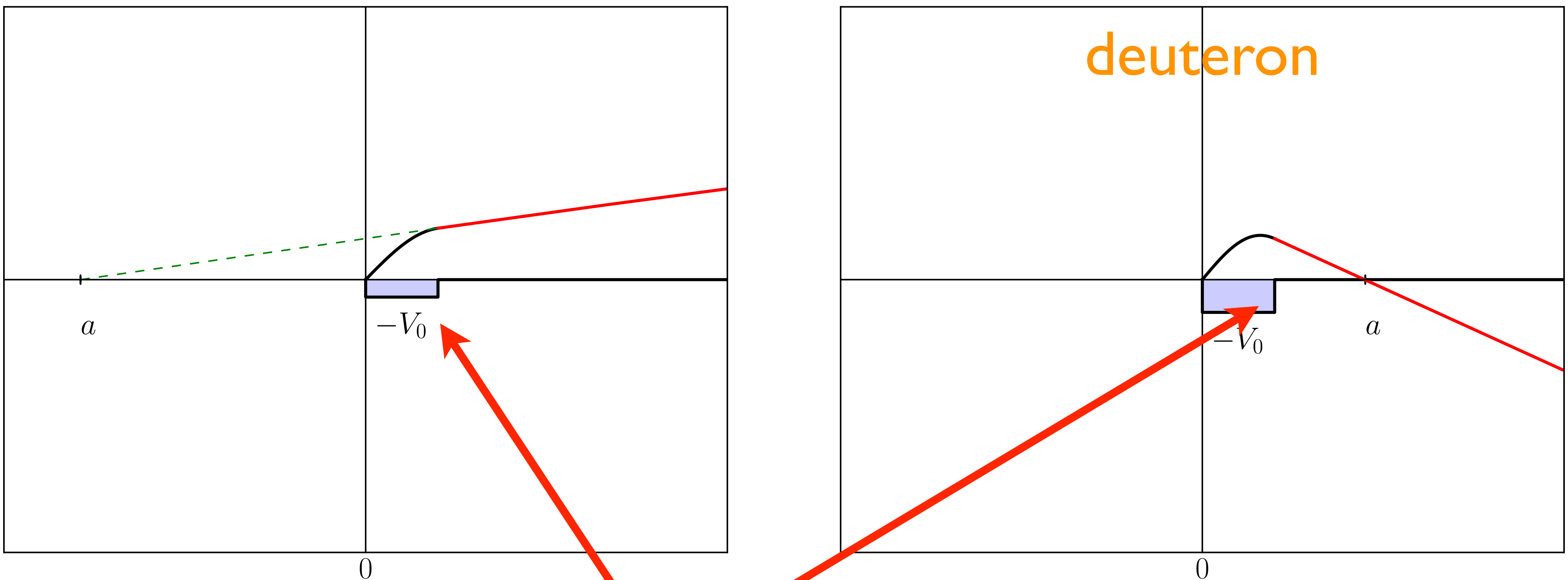
no stars like ours!

(also very sensitive to $m_n - m_p \propto \begin{cases} m_d - m_u \\ e^2/4\pi \end{cases}$)

we want to understand this from QCD

proton-neutron scattering at low energies

$^1S_0 : a \simeq -24 \ \texttt{fm}$ $\quad$ $^3S_1 : a \simeq 5.5 \ \texttt{fm}$



$$R_{NN} \sim 1.4 \ \texttt{fm}$$

Fine tuning gives small deuteron binding energy

Solving QCD can help us determine the nature of this fine tuning

Importance of QCD in Nuclear Physics

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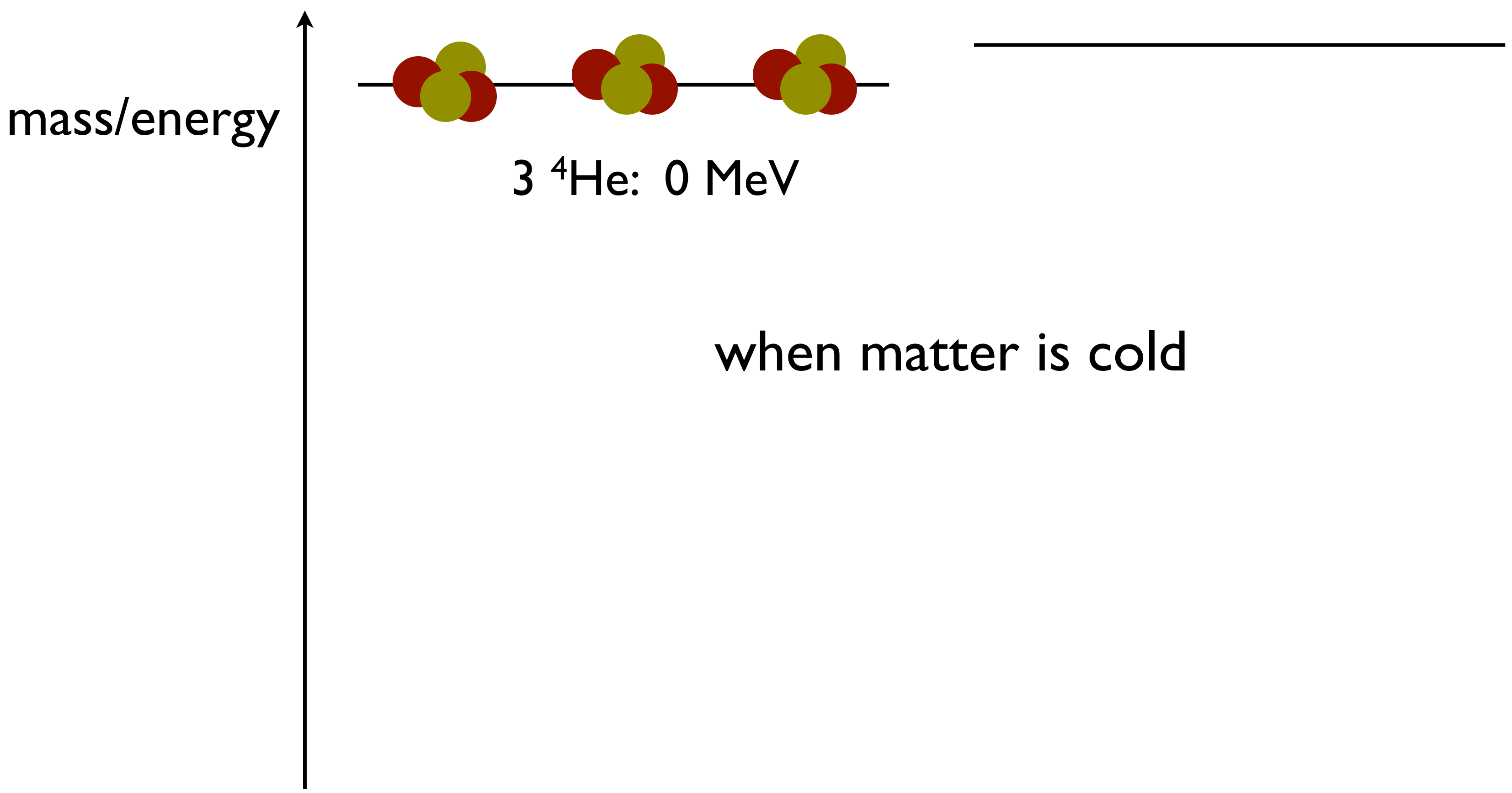
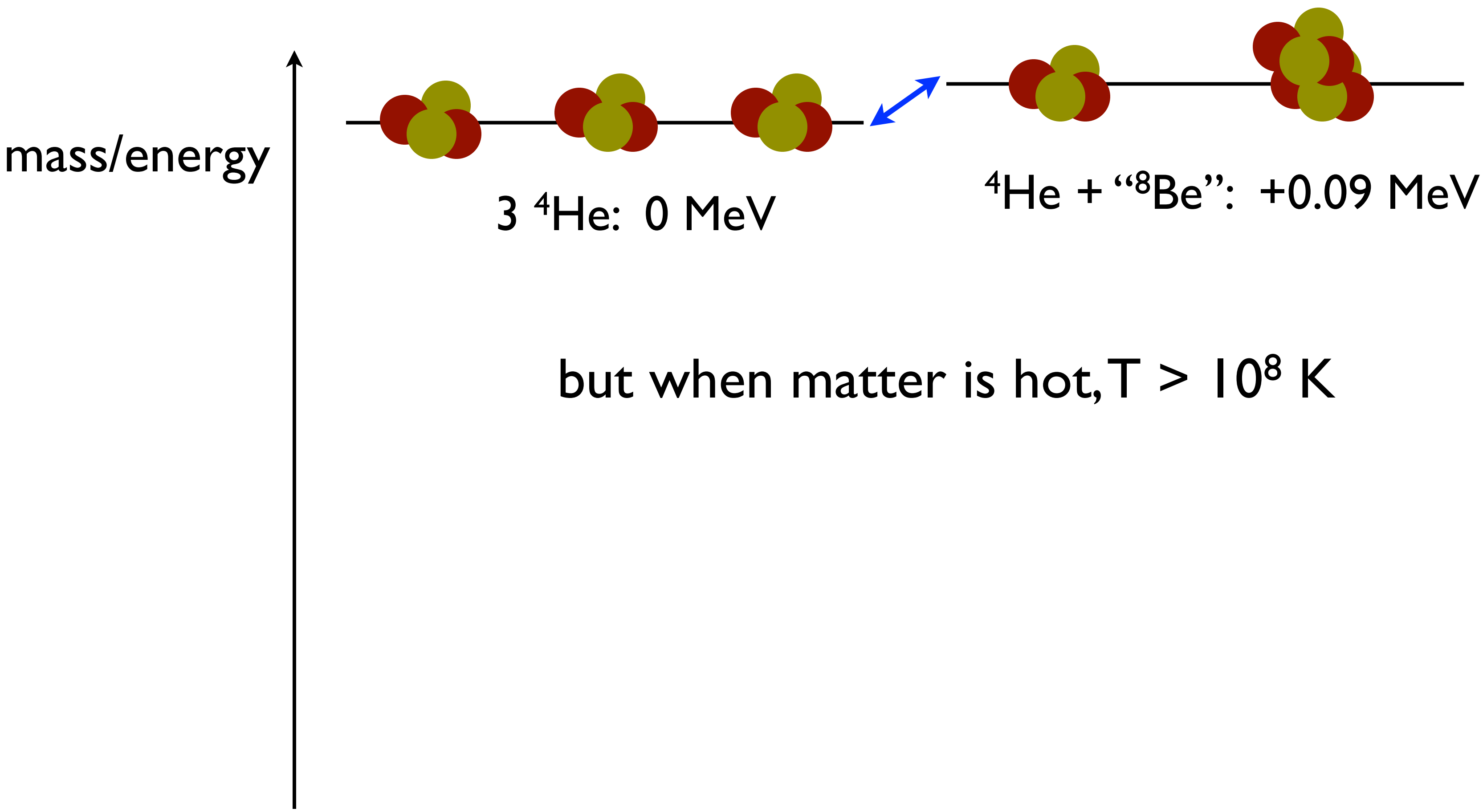
- ...

Large stars use He and neutrons to build new nuclei.

Higher temperatures and higher densities are needed.



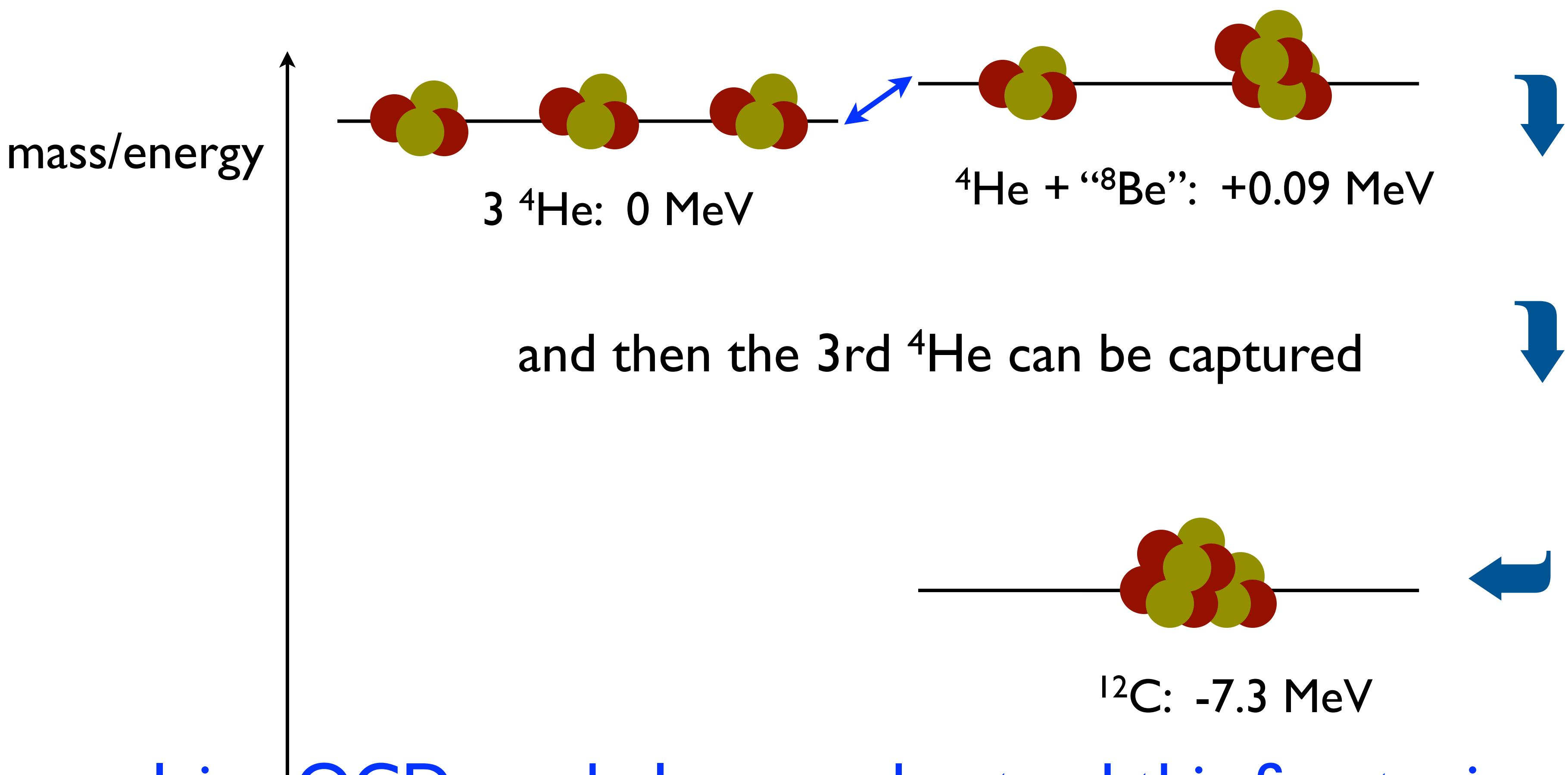
even more finely tuned



but when matter is hot, T > 10^8 K

even more finely tuned - source of complex life

the triple-α process Hoyle State



solving QCD can help us understand this fine tuning:

chance? fundamental?

Importance of QCD in Nuclear Physics

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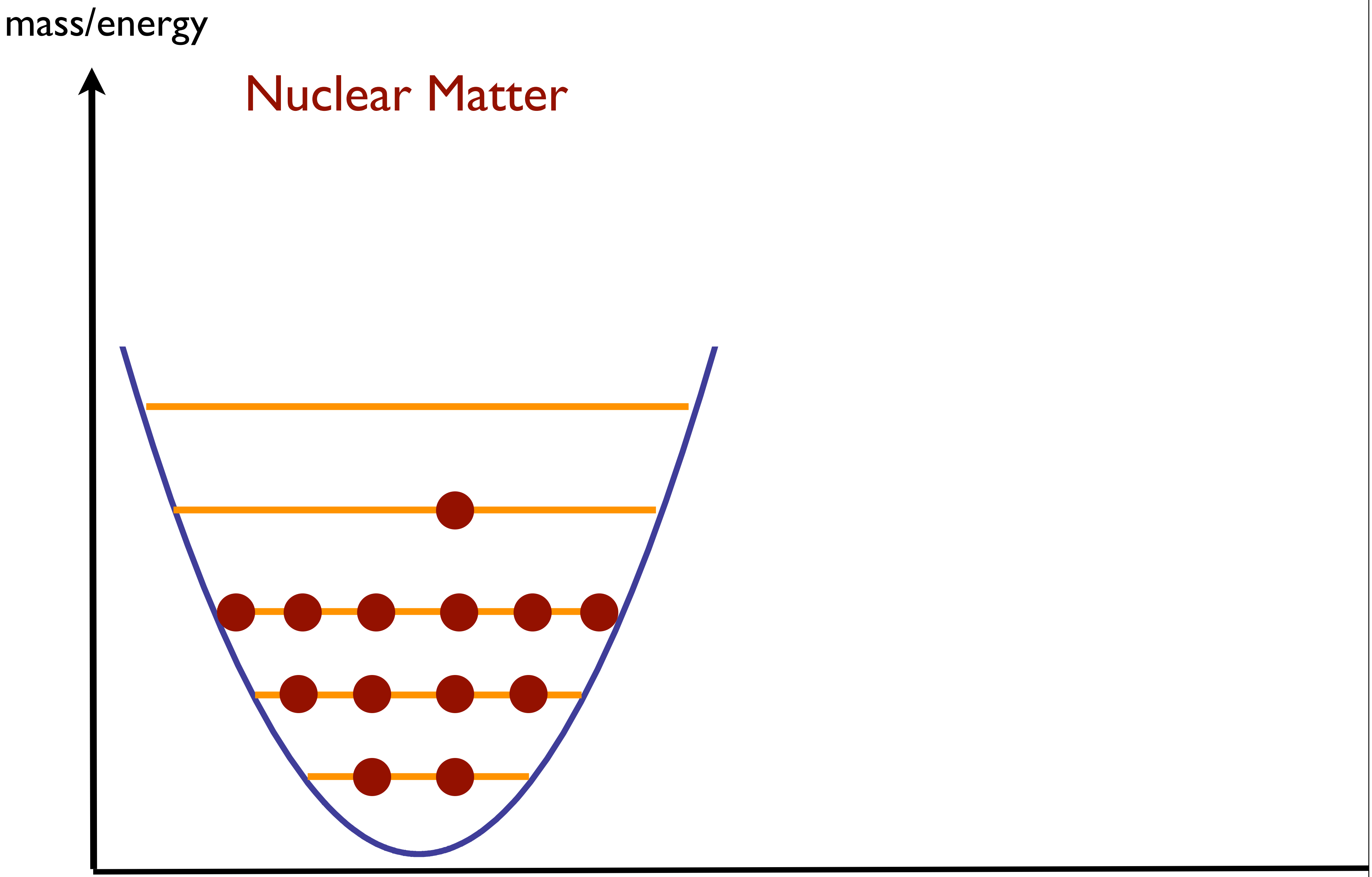
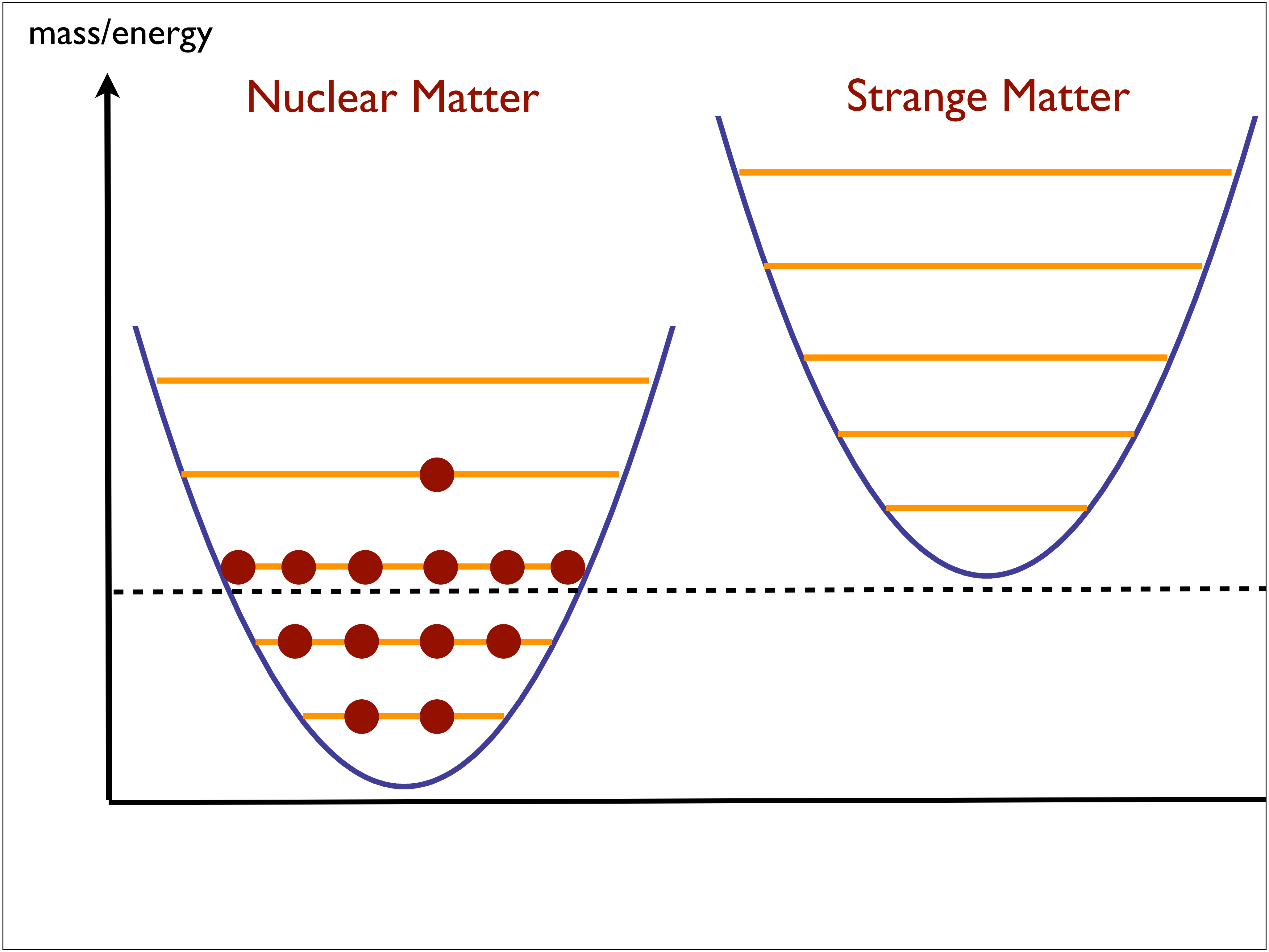
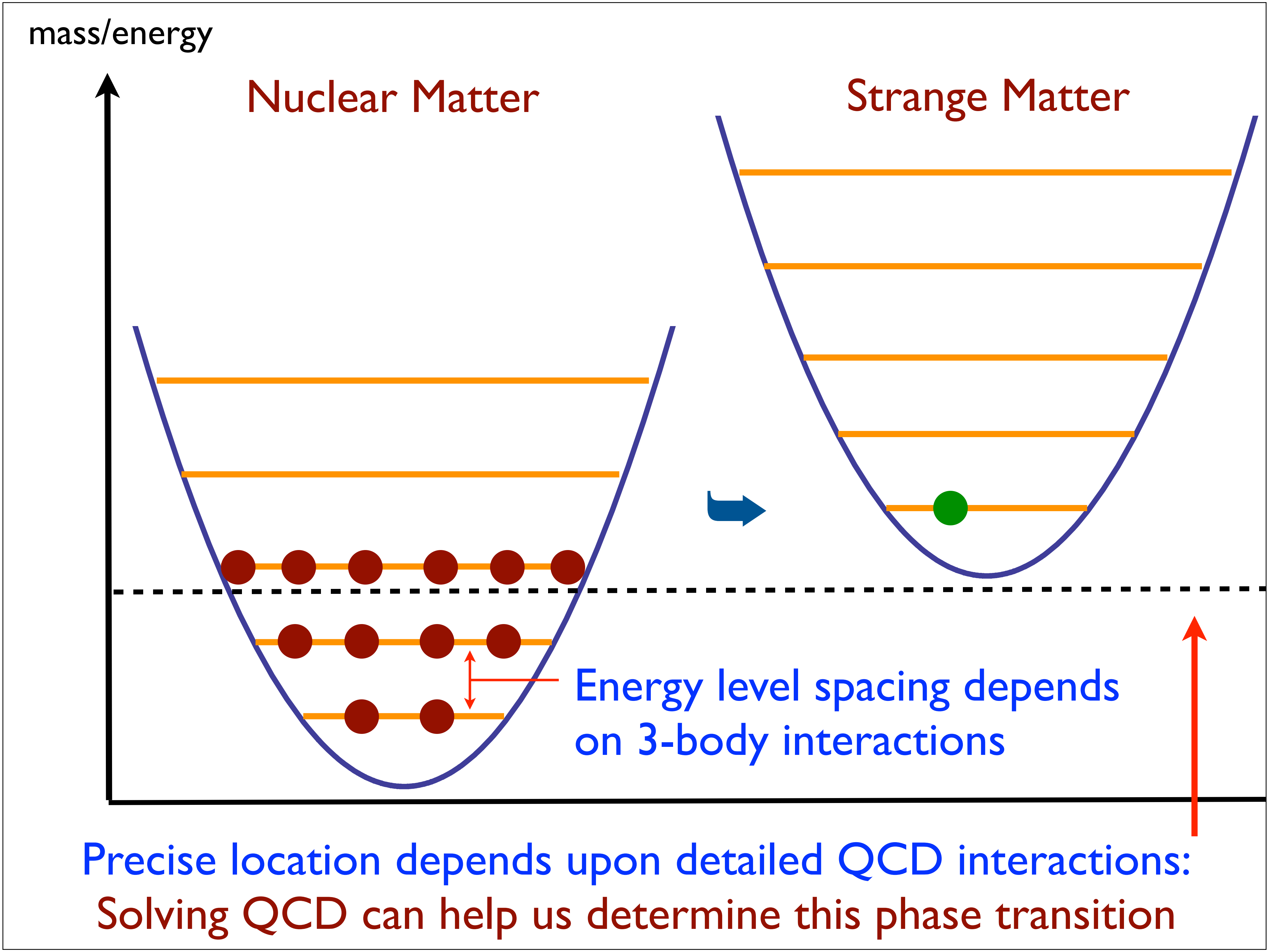

mass/energy
Nuclear Matter


mass/energy
Nuclear Matter
Strange Matter



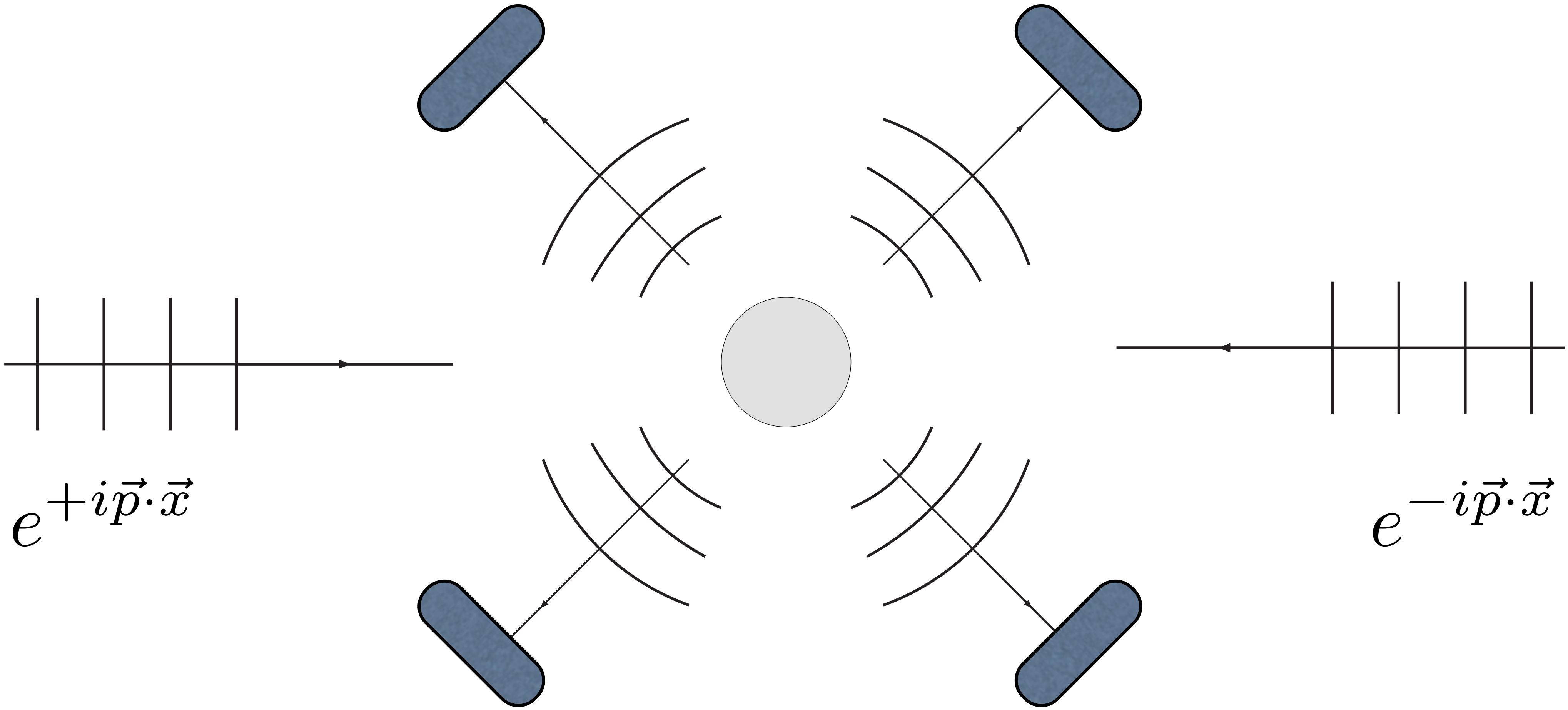
mass/energy
Nuclear Matter
Strange Matter
Energy level spacing depends
on 3-body interactions
Precise location depends upon detailed QCD interactions:
Solving QCD can help us determine this phase transition

Importance of QCD in Nuclear Physics

- Phase transition from QGP to Hadrons
- Big Bang Nucleosynthesis and primordial hydrogen and helium abundance
- Production of Carbon 12
- Equation of state of dense nuclear matter: neutron stars
- ... fundamental symmetries (parity violation, CP, ...)

Multi-Hadron Interactions:
Scattering

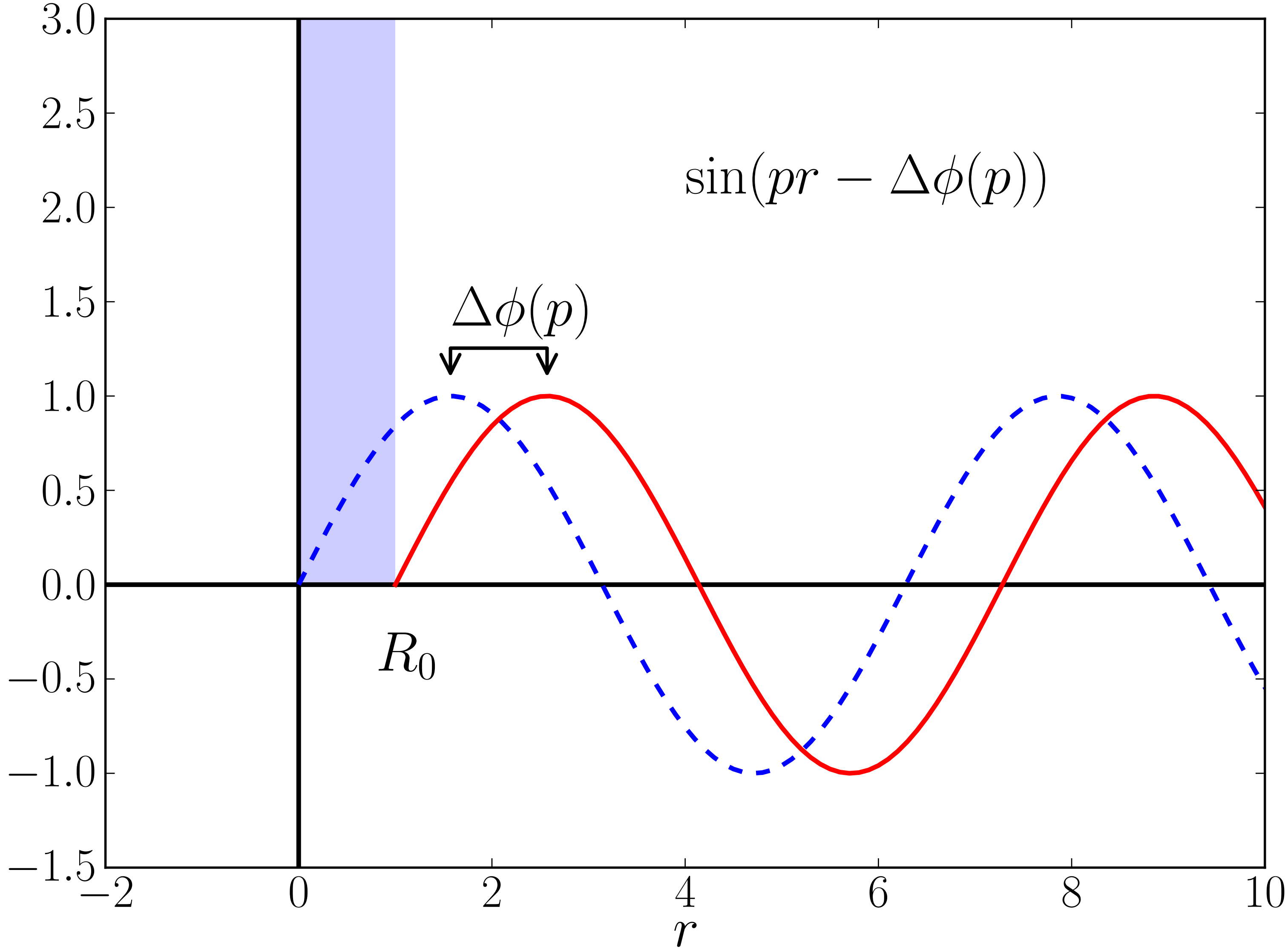
Scattering



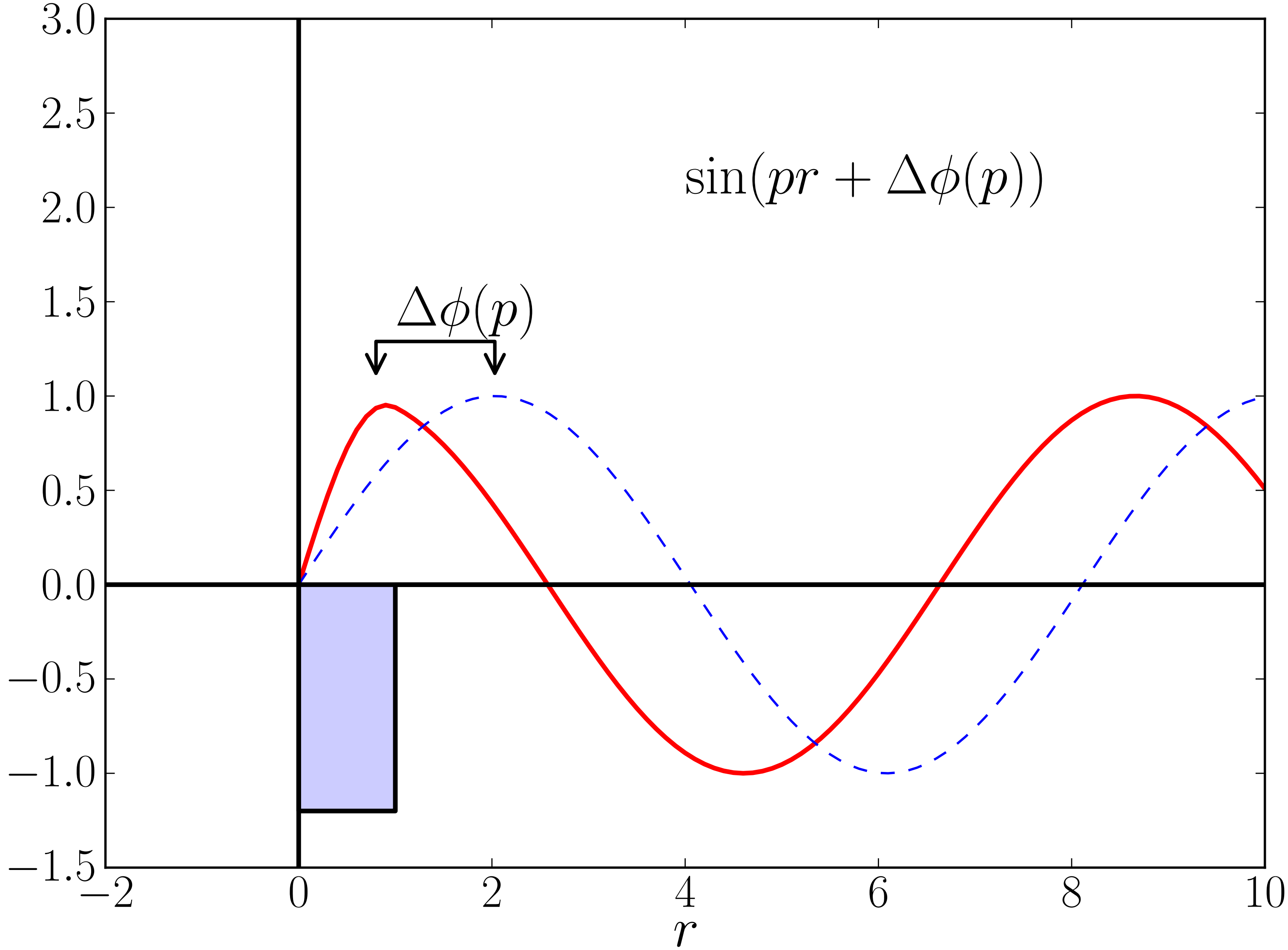
$$S_l(p) = e^{2i\delta_l(p)}$$

Scattering Phase Shift

Scattering off "Hard Sphere"

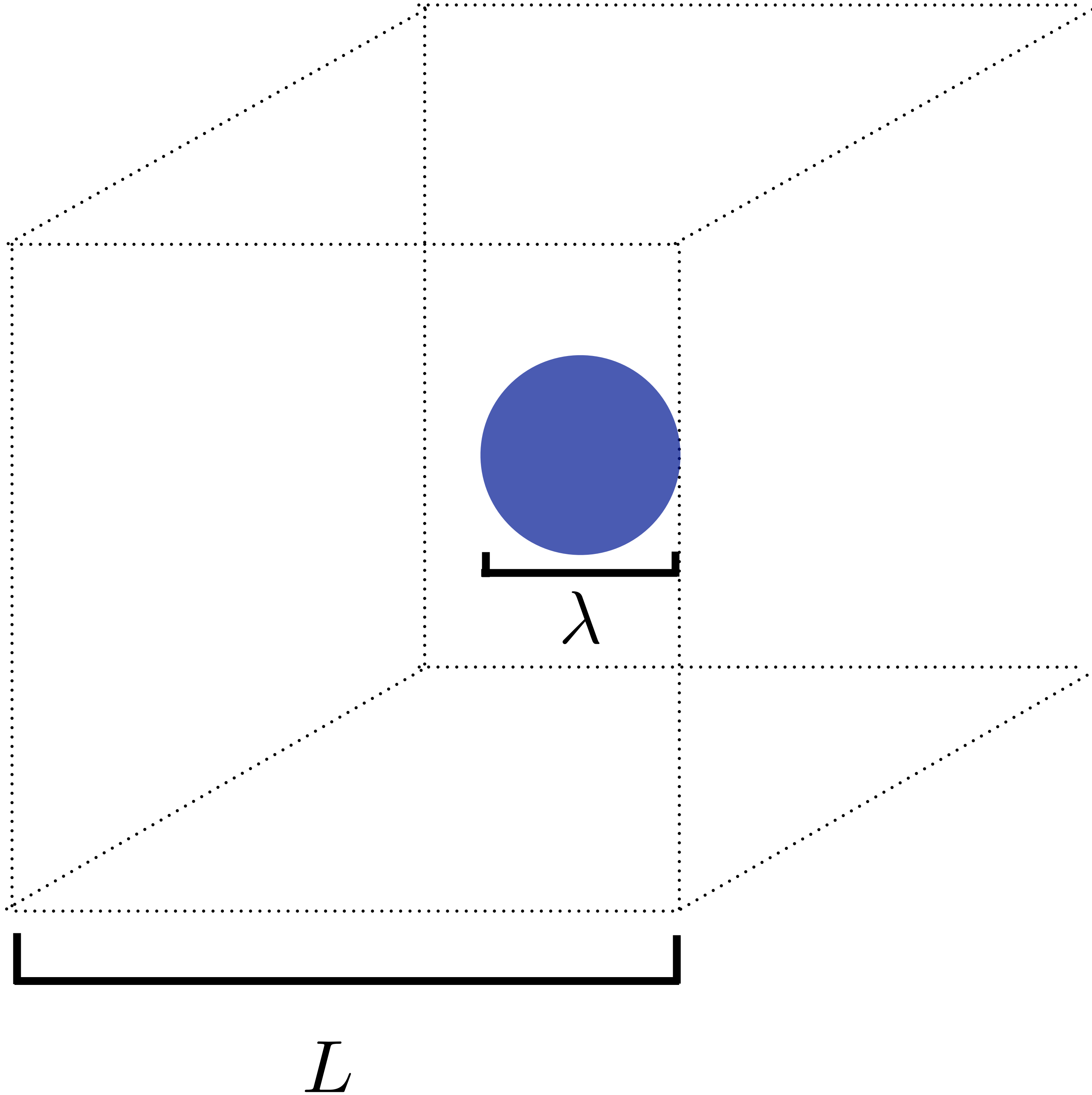


Scattering off "Soft Sphere"



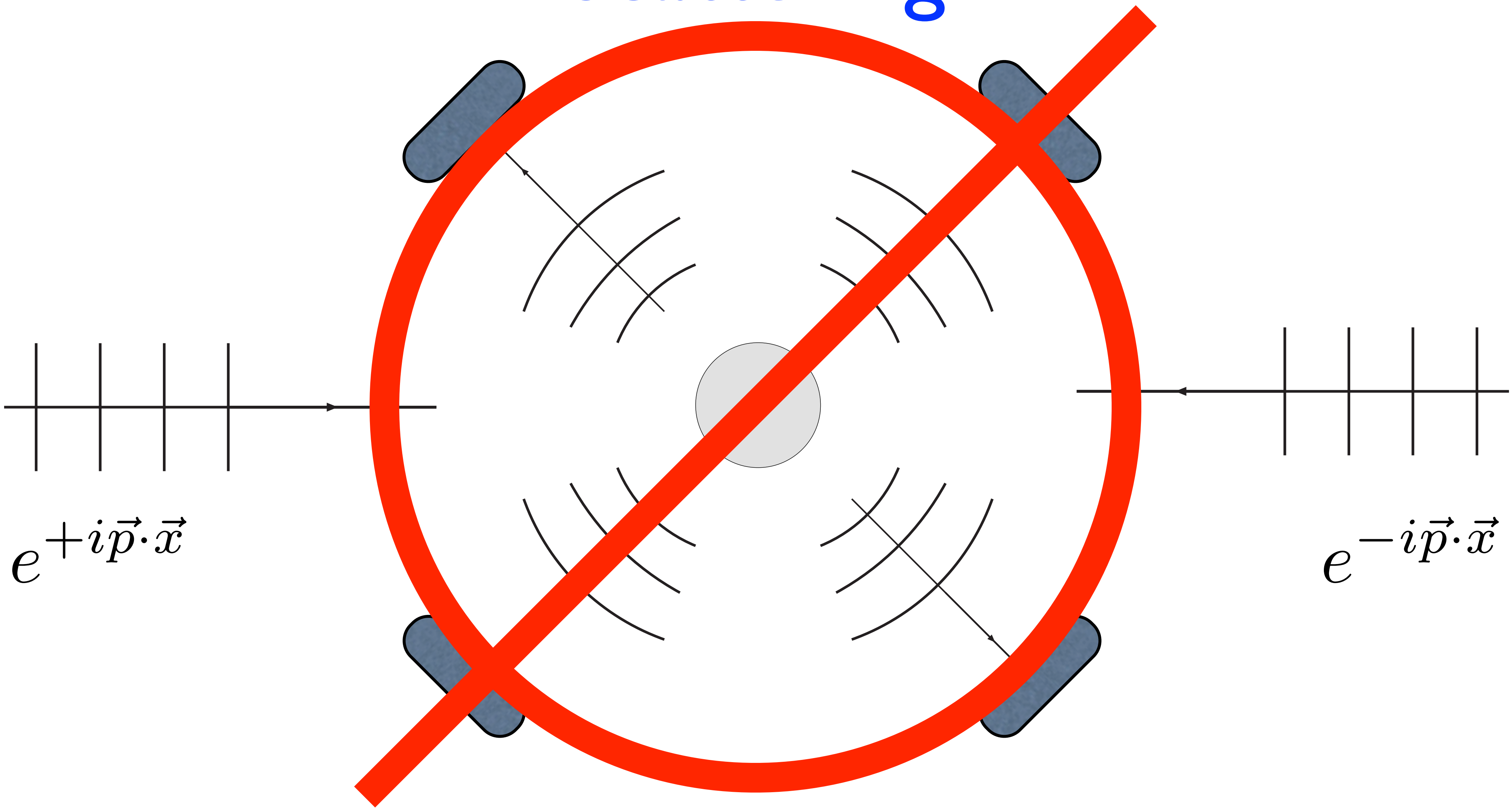
State of the art lattice QCD calculations

$$\frac{L}{\lambda} \sim 4 - 6$$



Pacman Boundary Conditions

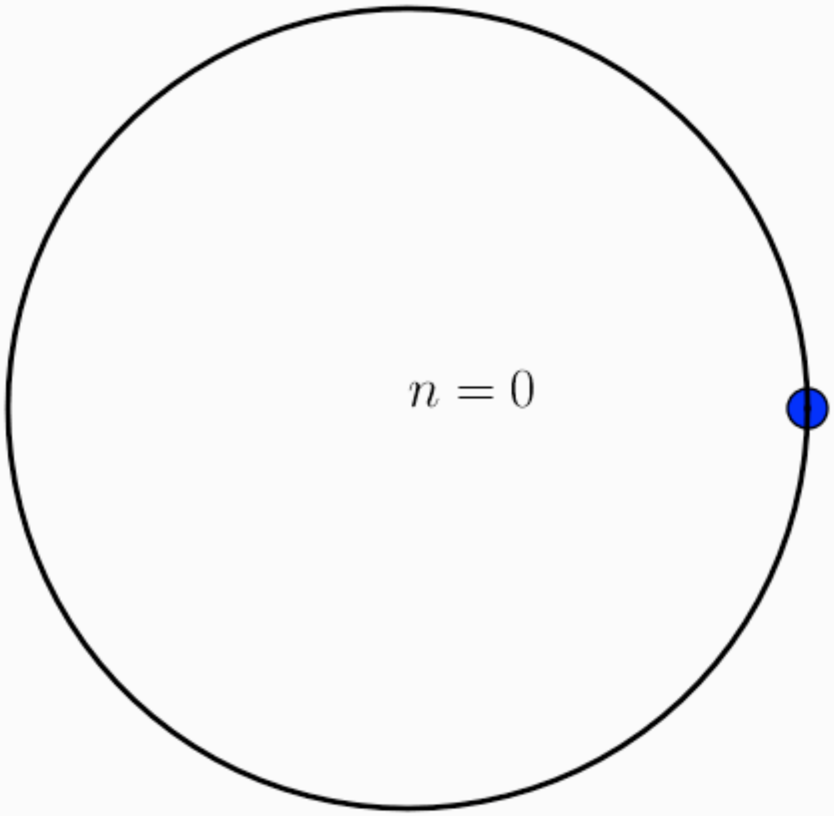
Scattering



$$S_l(p) = e^{2i\delta_l(p)}$$

Scattering Phase Shift

single particle cavity modes
one dimension

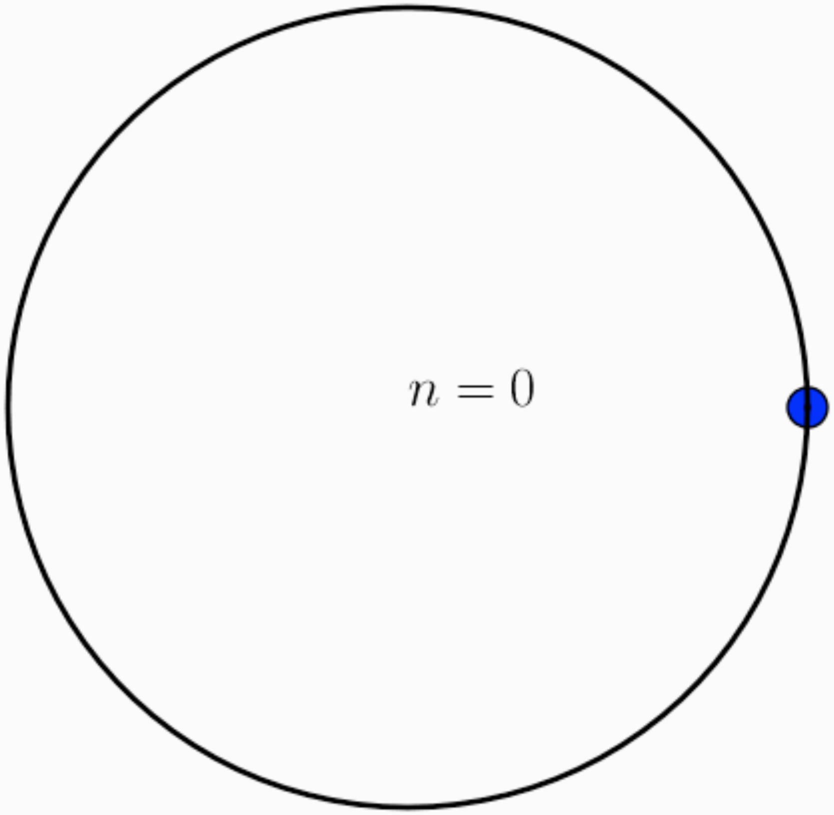


$$E_n = \sqrt{m^2 + q_n^2}$$

$$q_n = \frac{2\pi n}{L}$$

periodic boundary conditions

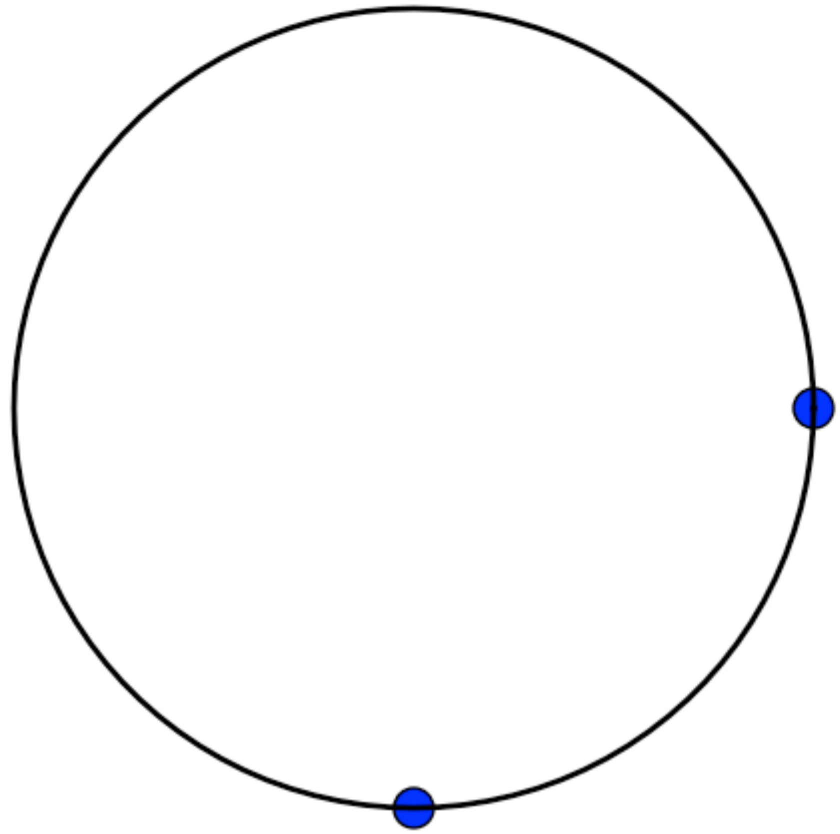
single particle cavity modes
one dimension



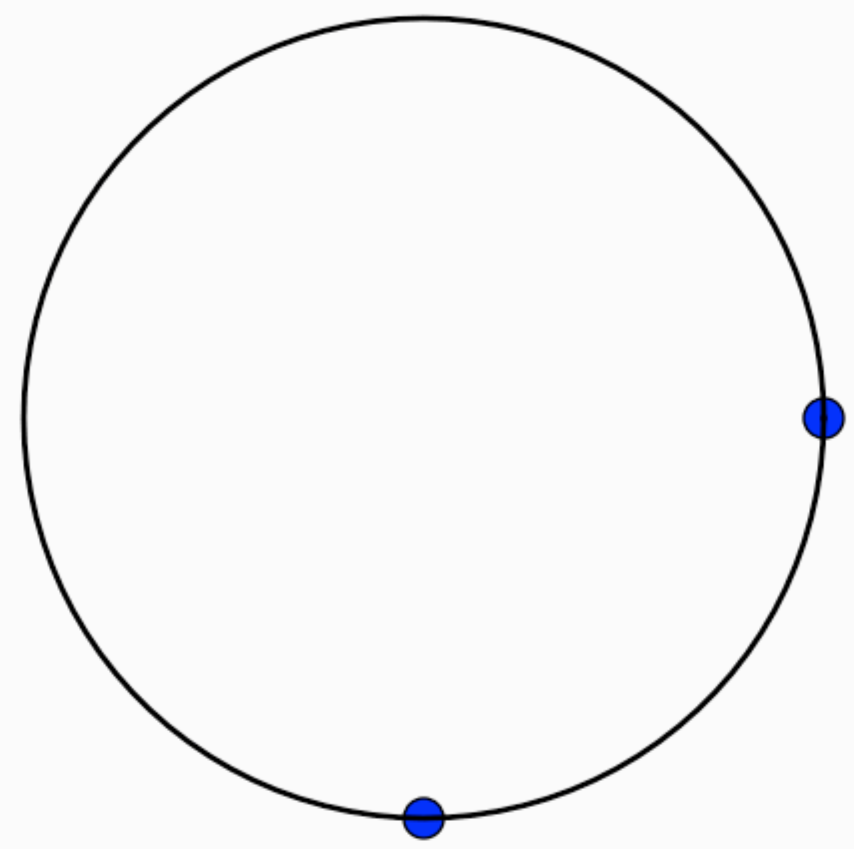
$$E_n = \sqrt{m^2 + q_n^2}$$
$$q_n = \frac{2\pi n}{L}$$

periodic boundary conditions

two particle modes

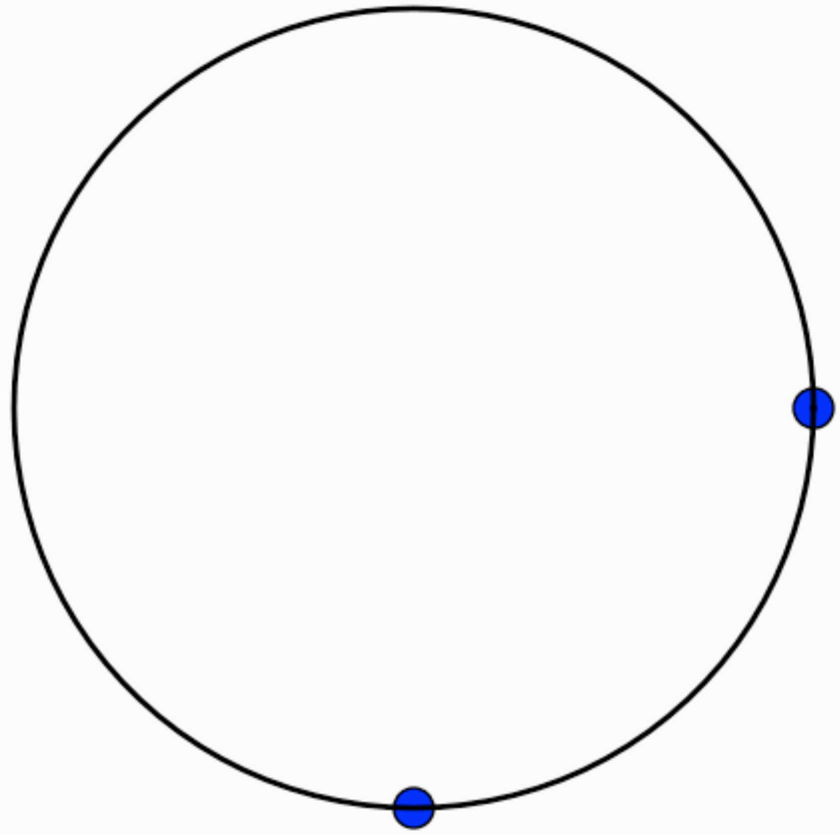


two particle modes



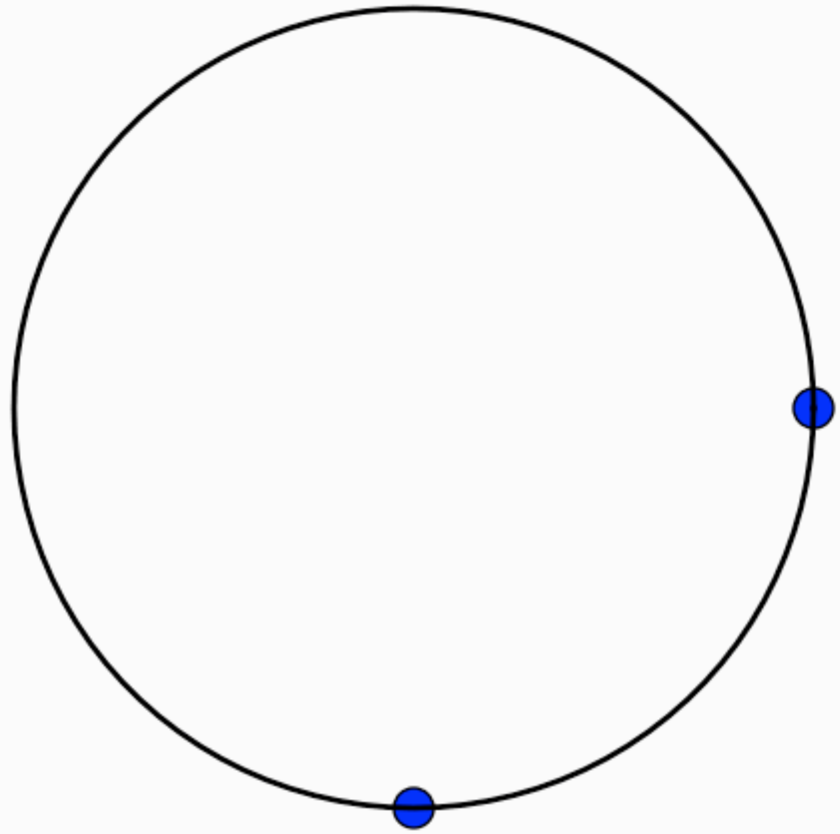
repulsive interaction

two particle modes



attractive interaction

two particle modes



stronger attraction

two particle modes

energy eigenvalues modified by interactions

$$E_{(n)} = 2m + \Delta E_{(n)} = 2\sqrt{m^2 + p_{(n)}^2}$$

LQCD

absence of interactions

$$p_{(n)} = q_n = \frac{2\pi n}{L}$$

two particle modes

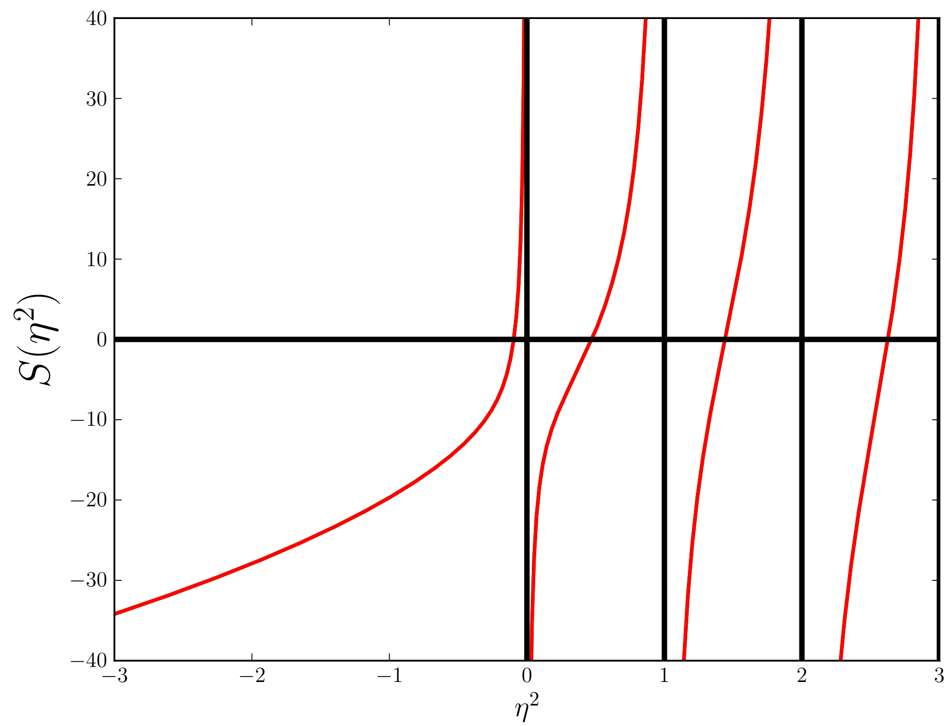
energy eigenvalues modified by interactions

$$E_{(n)} = 2m + \Delta E_{(n)} = 2\sqrt{m^2 + p_{(n)}^2}$$

rigorous relation

$$\delta(p) = \cot^{-1}\left[\frac{1}{\pi p L} S\left(\left(\frac{pL}{2\pi}\right)^2\right)\right]$$

knowing the phase shift is equivalent to knowing the two-particle interactions



$$S(\eta^2) = lim_{\Lambda\to\infty} \sum_{\mathbf{n}}^{\Lambda} \frac{1}{\mathbf{n}^2 - \eta^2} - 4\pi\Lambda$$

Challenges of Lattice QCD?

- standard challenges for lattice QCD
- challenges for nuclear physics applications of lattice QCD

Lattice QCD: Standard Challenges

- lattice spacing: desire at least 3 lattice spacings (preferably all < 0.1 fm)

$$t_{cpu} \sim 1/a^6$$

- lattice volume: $m_\pi L \geq 4$ (simple quantities)

  $m_\pi L \geq 2\pi$ better

- quark mass: desire to run at physical quark masses (even better $100 \lesssim m_\pi \lesssim 300 \text{ MeV}$)

$$t_{cpu} \sim 1/m_q$$

- disconnected diagrams: computationally much more expensive both in cpu hours and file storage

Lattice QCD: Challenges for Nuclear Phsysics

- energy scales: energy scales of interest to nuclear physics are MeV (or even KeV) while total energy is GeV

$$\gamma = \sqrt{MB} \qquad \gamma_{deut} \simeq 45 \text{ MeV}$$

new small scale

$$m_\pi L >> 1 \qquad \gamma L >> 1$$

- signal to noise problem: baryon correlation functions have exponentially hard signal to noise problem

$$S/N \sim \mathcal{Z} e^{-A(m_N - \frac{3}{2} m_\pi)t} \quad A = \text{ number of nucleons}$$

- large basis of interpolating fields: to project onto the various densely packed energy levels in (small) nuclei, need large basis of operators

- Wick contractions: all the Wick contractions of the quark fields must be formed

these are/will be the dominant cost for the entire calculation

Computational Cost

For serious, respectable, calculation of nucleon-nucleon interactions, at the physical pion mass, with existing algorithms

$a \simeq 0.13\ \mathtt{fm}$ $\qquad m_\pi \simeq 140\ \mathtt{MeV}$ $\qquad L \simeq 8\ \mathtt{fm}$

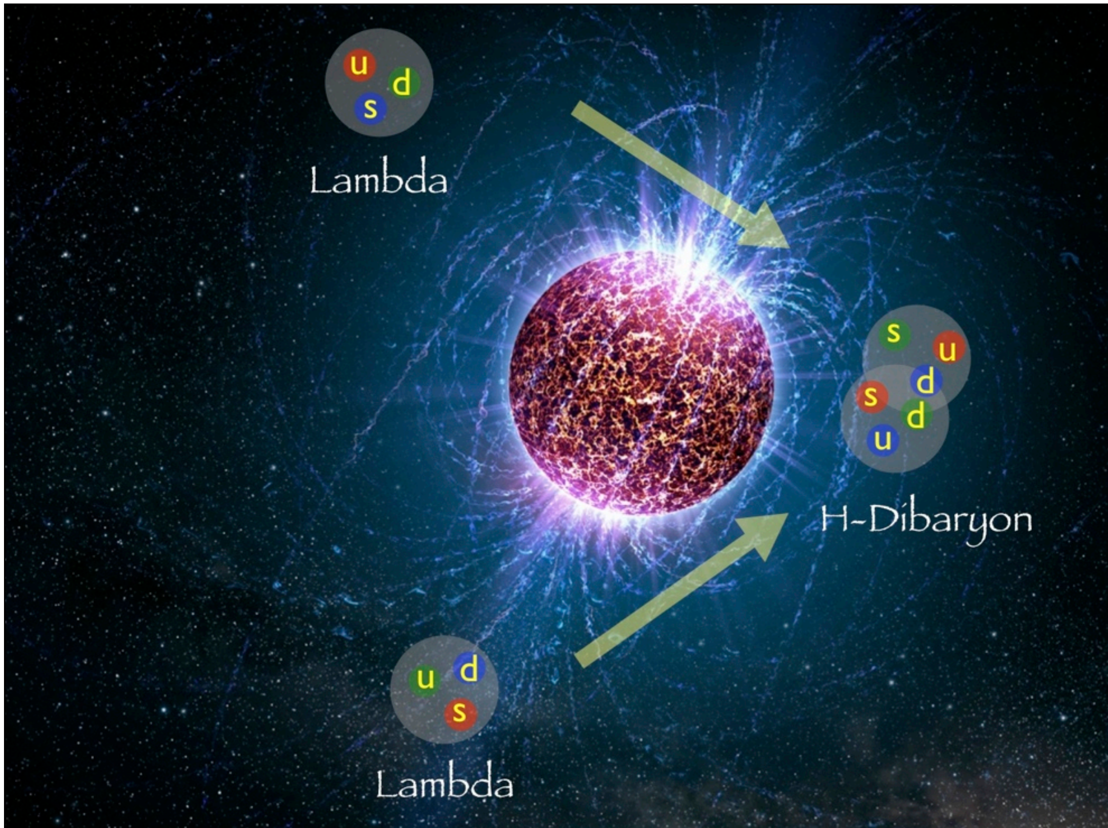
$$m_\pi L \simeq 5.5$$

$$t_{CPU} \sim 100\ \text{TeraFlops Years!}$$

1 Billion CPU hours

~1 week on Sequoia (20 PetaFlops) BG/Q @ LLNL

Examples



H-Dibaryon

$$|B\rangle \sim |uds\rangle$$

$$|H\rangle \sim |uuddss\rangle$$

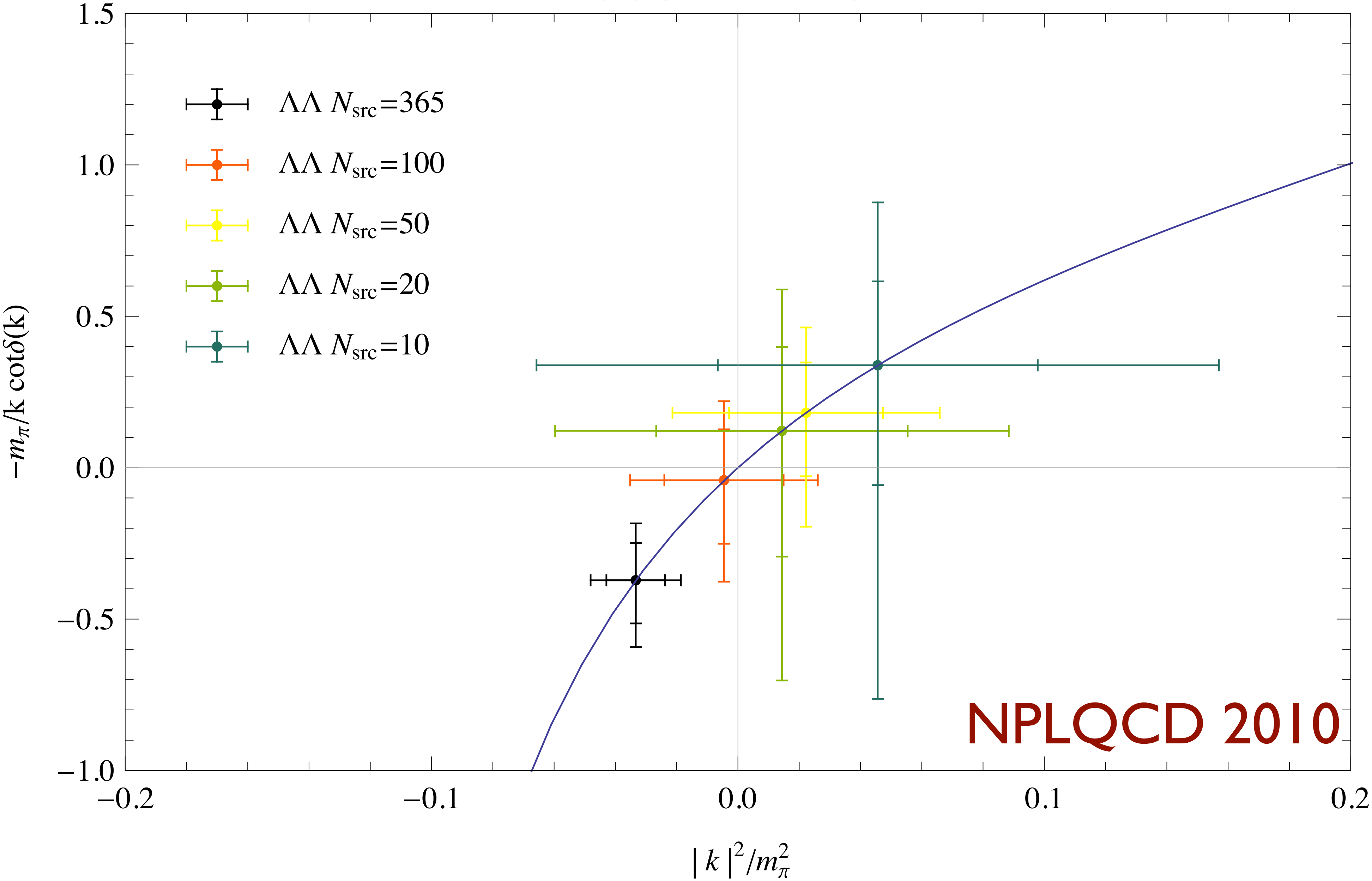
$$\sim |\Lambda\Lambda\rangle$$

1977 - Jaffe proposed H-dibaryon bound state

In dense nuclear matter, energetically favorable

Bound H-Dibaryon not found experimentally, but evidence for shallow bound state or resonance

High statistics crucial
~0.5 x 10^6



Is H-Dibaryon bound?

$$\Delta E = 2\sqrt{M^2 + k^2} - 2m$$

Scattering State: $$\Delta E = \frac{4\pi a}{ML^3}\left[1 + \mathcal{O}\left(\frac{a}{L}\right)\right]$$

Bound State: $$\kappa^2 = -k^2$$

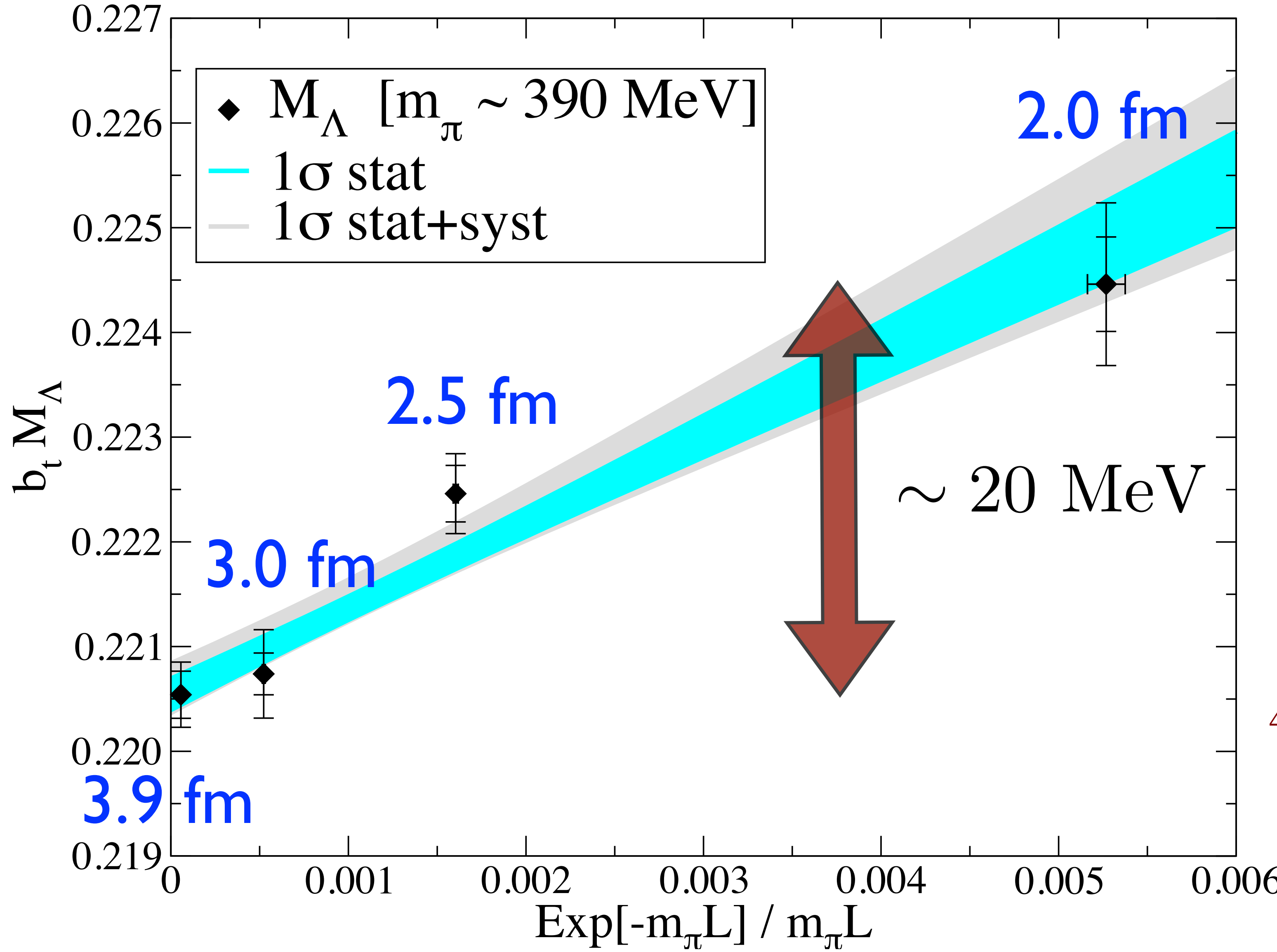
$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right) + \cdots$$

$$\gamma = \sqrt{M_\Lambda^\infty B_H^\infty}$$

Need Multiple Volumes! (or momentum boosted systems)

Volume dependence of Lambda

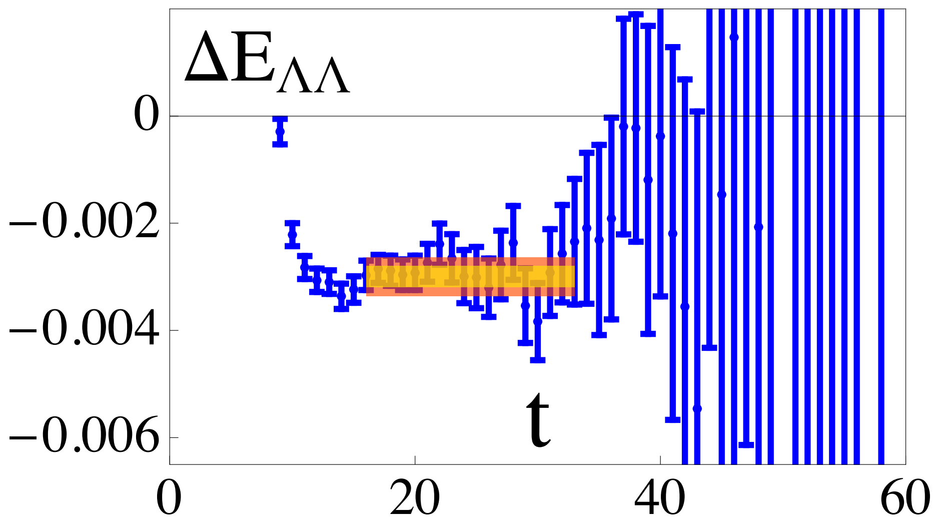
NPLQCD: PRD 84 (2011)



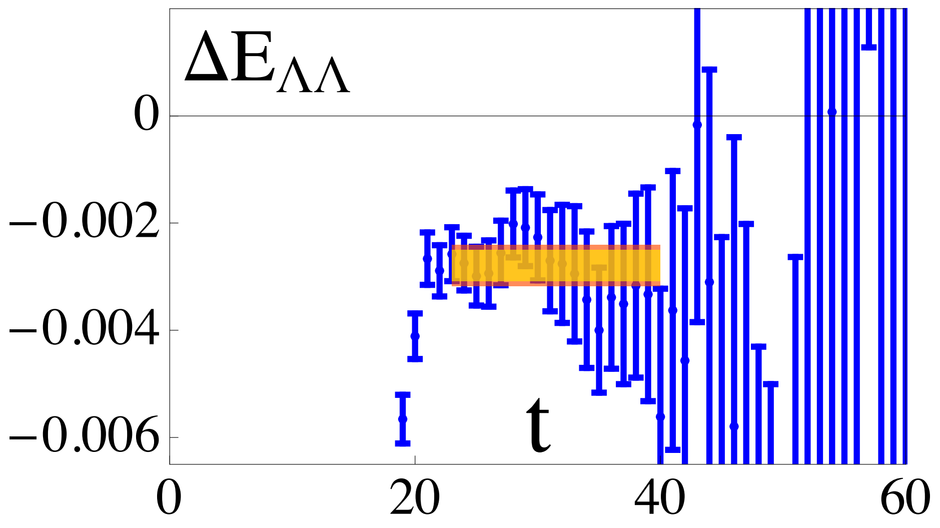
$m_\pi \simeq$ 400 MeV

$m_\pi L =$ 7.7, 5.8 4.8 3.9

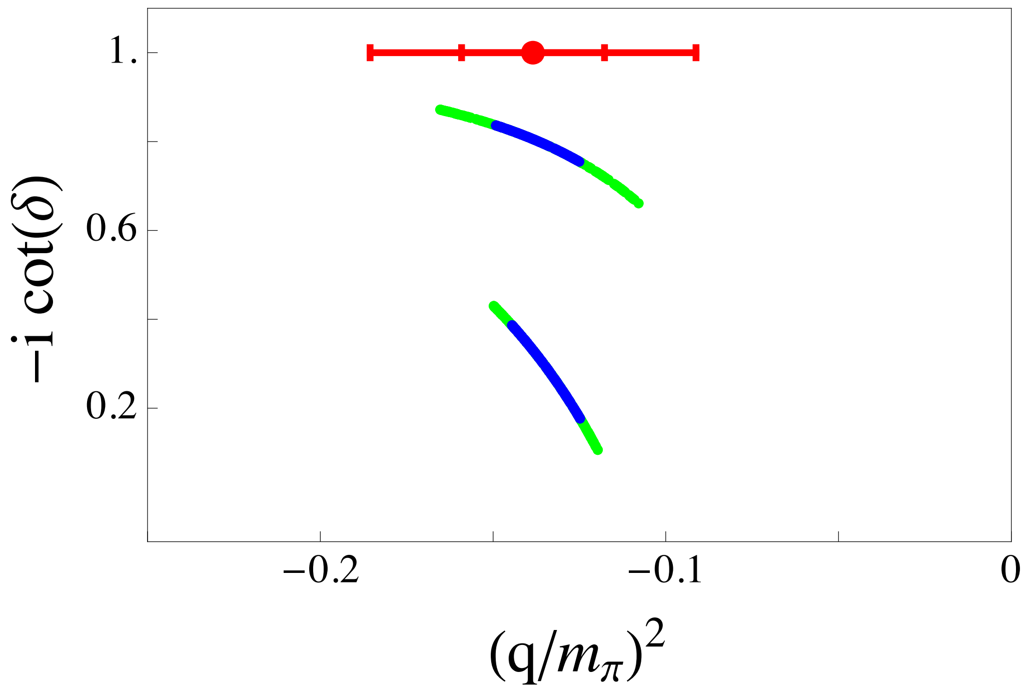
NPLQCD: PRL 106 (2011)



$L \simeq 3.0$ fm



$L \simeq 3.9$ fm



$$B = 16.6 \pm 2.1 \pm 4.6 \text{ MeV}$$

(no electro-weak, single lattice spacing, single pion mass)

$$m_\pi \sim 400 \text{ MeV}$$

H-Dibaryon from Lattice QCD

- "Evidence for a bound H-Dibaryon from lattice QCD" PRL 106, 162001 (2011)

$$N_f = 2+1, \;\; a_s \simeq 0.12 \text{ fm},$$
$$m_\pi \simeq 390 \text{ MeV}, \;\; L = 2.0, 2.5, 3.0, 3.9 \text{ fm}$$

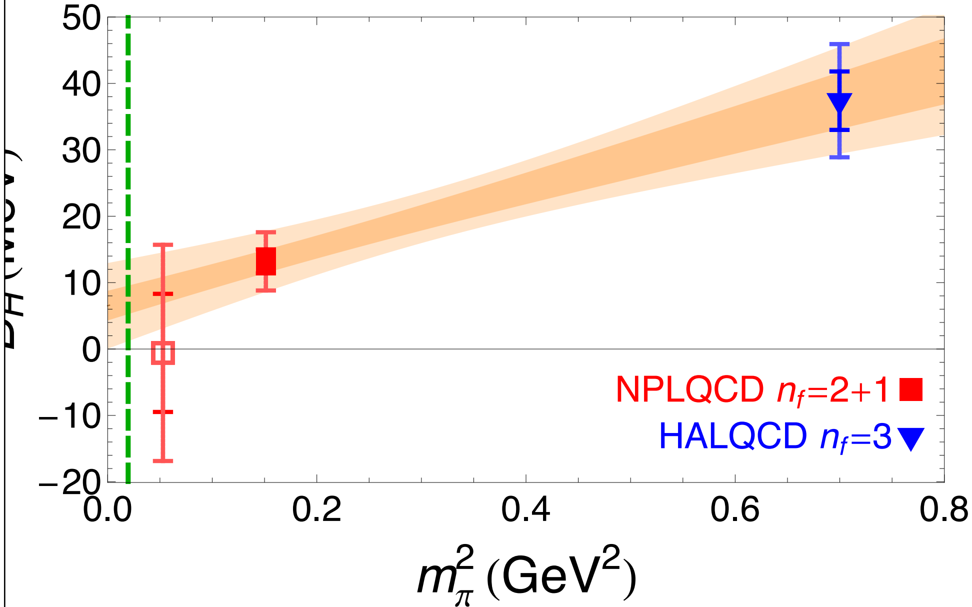
- "Bound H-dibaryon in flavor SU(3) limit of lattice QCD" PRL 106, 162002 (2011)

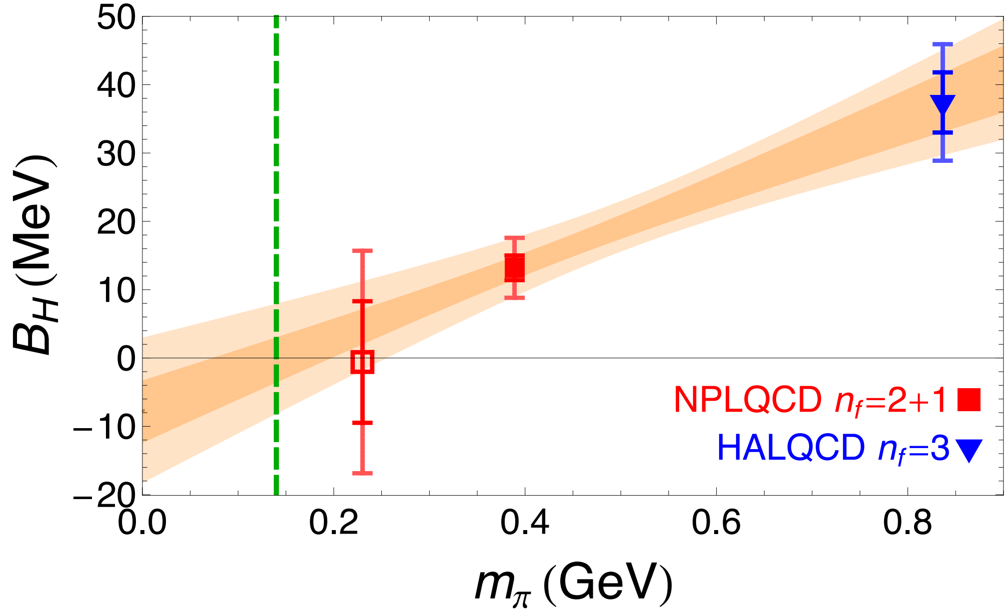
$$N_f = 3, \;\; a_s \simeq 0.12 \text{ fm},$$
$$m_\pi \simeq 670, 830, 1015 \text{ MeV}, \;\; L = 2.0, 3.0, 3.9 \text{ fm}$$

But not observed experimentally!

Simple extrapolation to physical pion mass

NPLQCD: Mod.Phys.Lett. A 26 (2011)





More sophisticated extrapolations (effective field theory) consistent with linear in pion mass

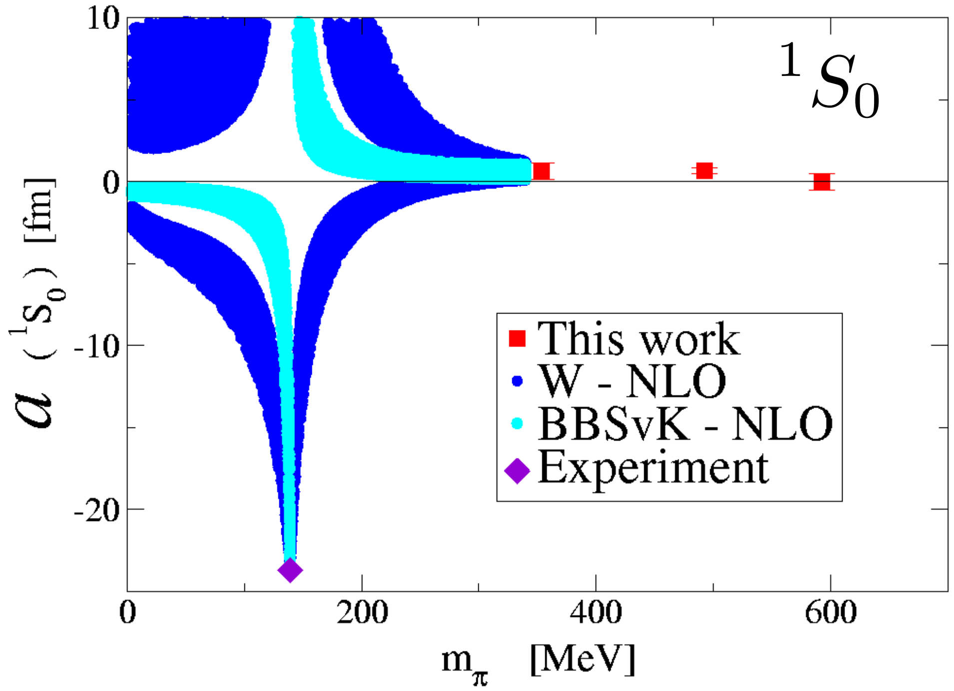
Haidenbauer and Meissner: Phys.Lett. B 706 (2011)

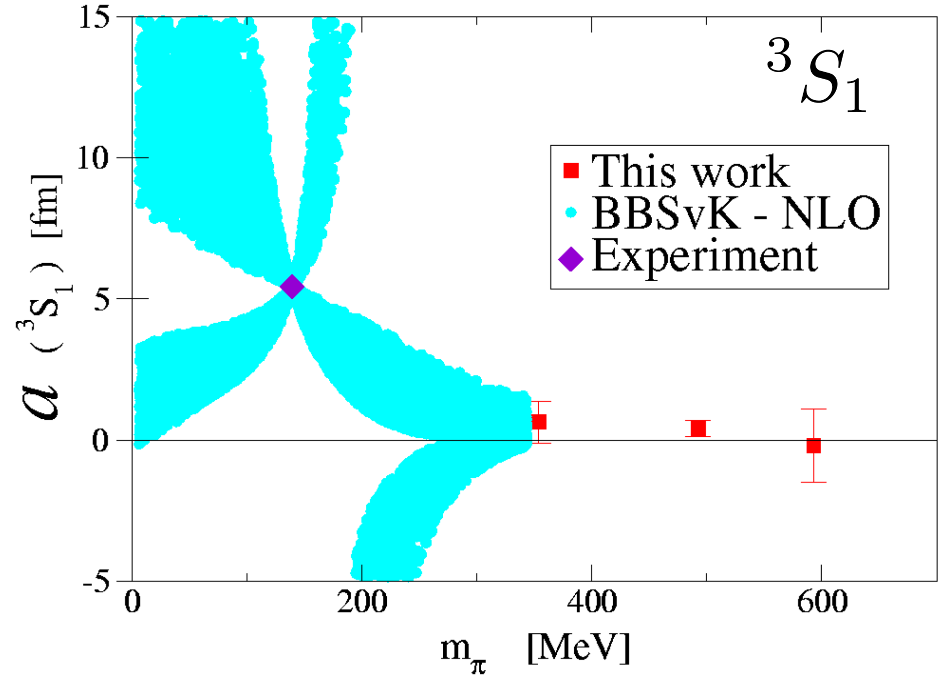
Haidenbauer and Meissner: arXiv:1111.4069

electromagnetic and weak interactions not included

Nucleon-Nucleon Interactions

Lattice QCD 2006: NN scattering





For quark masses > 2.5 x physical:

fine tuning gone

Nucleon-Nucleon

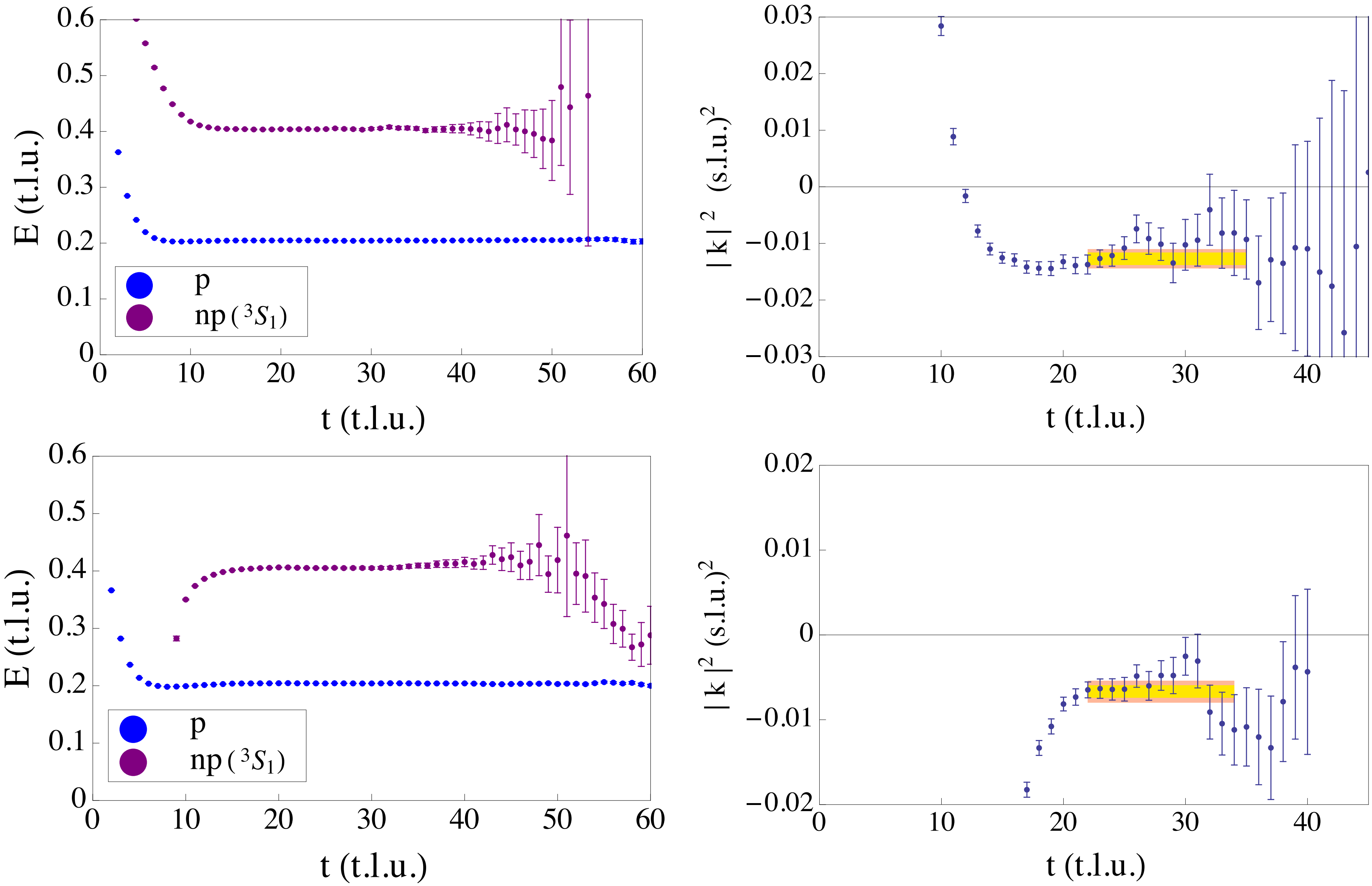
Deuteron

NPLQCD: PRD 85 (2012)

L[fm]

~ 3

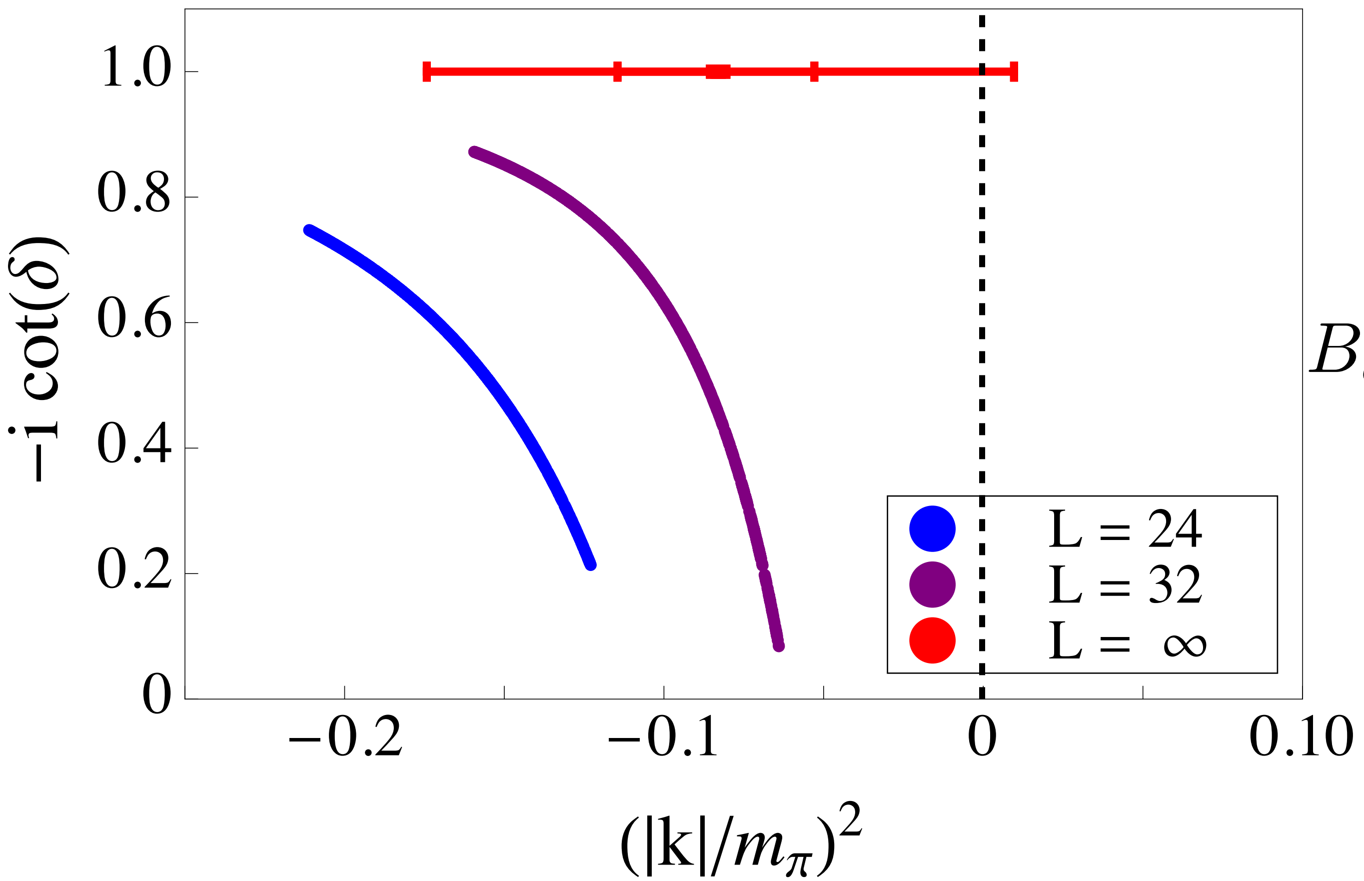
~ 4



Nucleon-Nucleon

NPLQCD:
PRD 85 (2012)

Deuteron



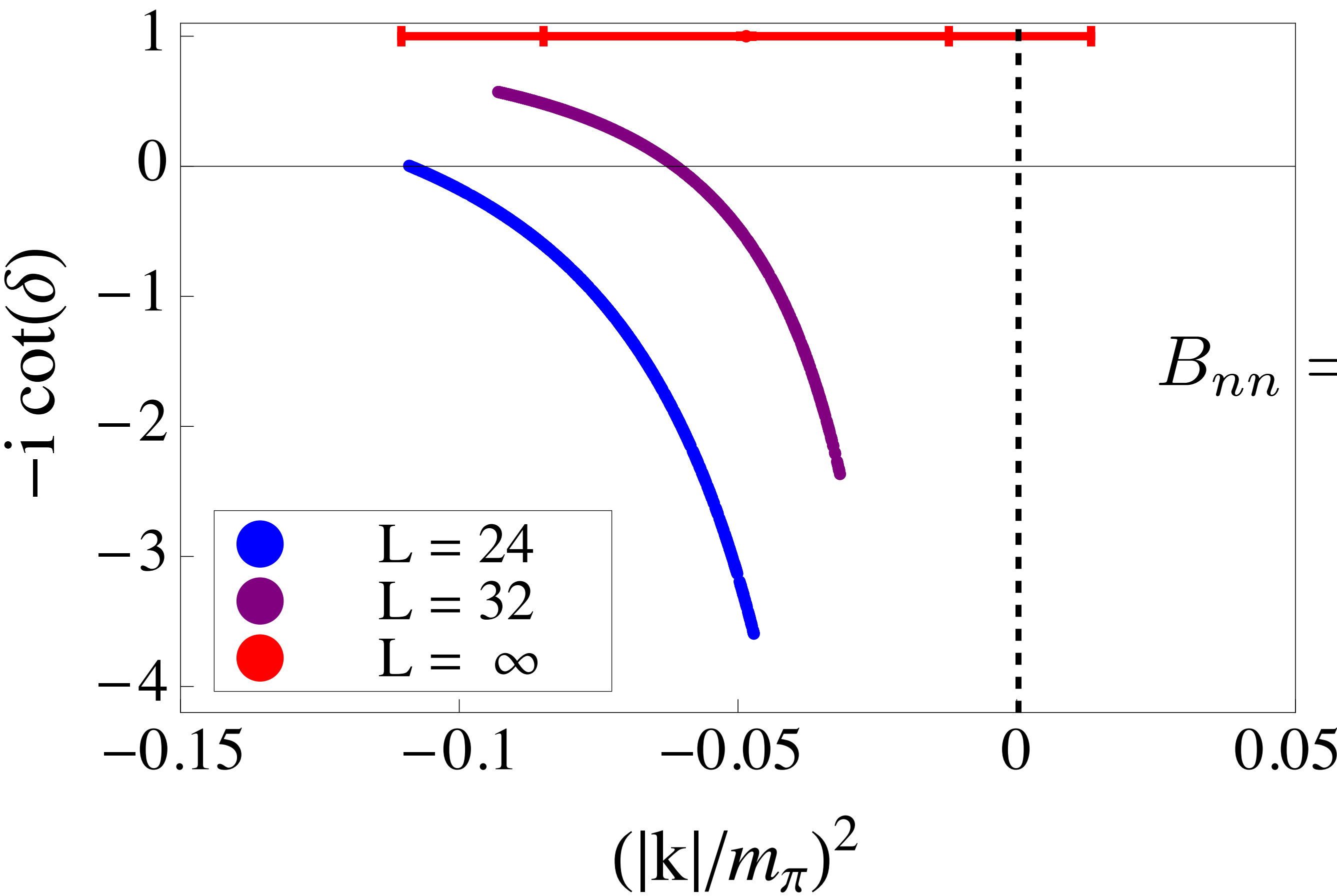
$$B_d = 11 \pm 5 \pm 12 \text{ MeV}$$

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right) + \cdots$$

Nucleon-Nucleon

Di-Neutron

NPLQCD:
PRD 85 (2012)

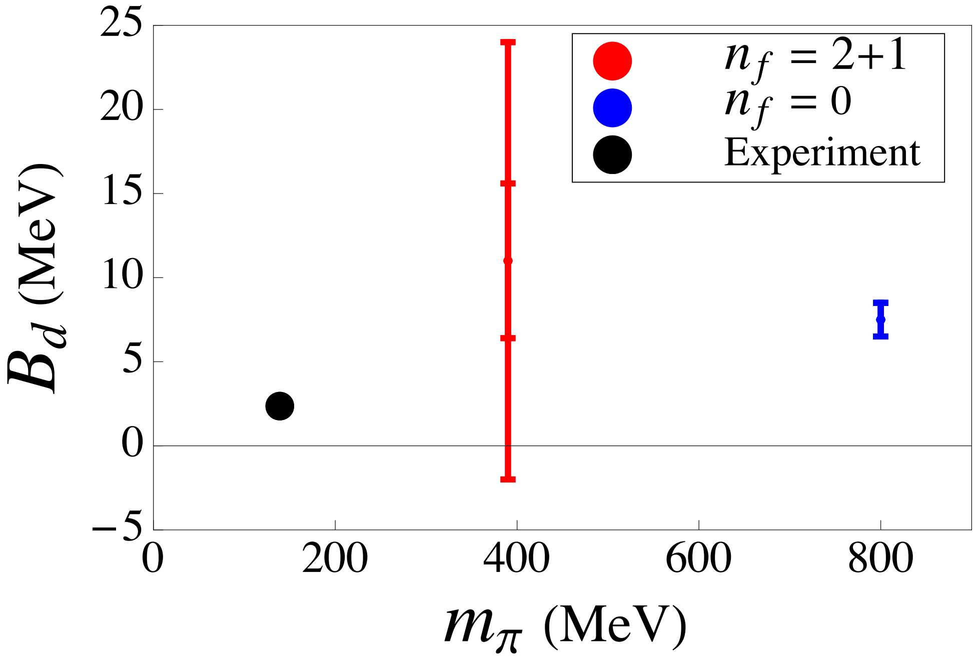


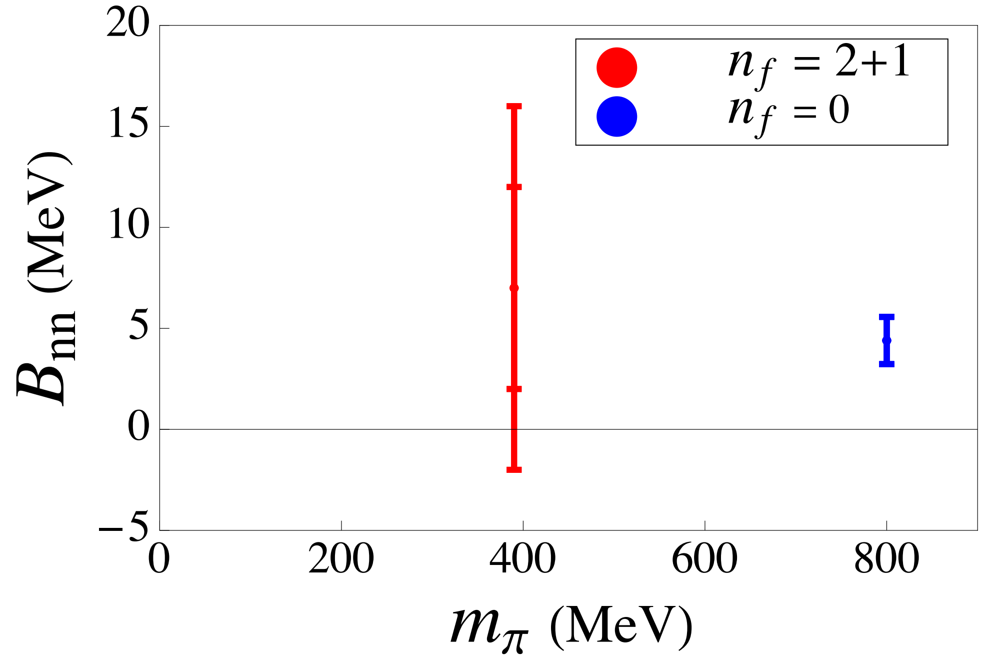
$$B_{nn} = 7.1 \pm 5.2 \pm 7.3 \text{ MeV}$$

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right) + \cdots$$

Nucleon-Nucleon

NPLQCD:
PRD 85 (2012)





Bound deuteron at this pion mass was not expected by most

$n_f=0$:
Yamazaki, Kurumashi, Ukawa: PRD 84 (2011)

Hyperon-Nucleon Interactions
and
Neutron Stars

Approximate neutron star by sea of static neutrons

Fumi's Theorem

Change in energy due to impurity

$$\Delta E = -\frac{1}{\pi\mu}\int_0^{k_F} dkk \sum \delta_l(k)$$

add Sigma Hyperons

$\rho_n \sim 0.4 \text{ fm}^{-3}$

$$\mu_n \simeq M_n + 150 \text{ MeV}$$

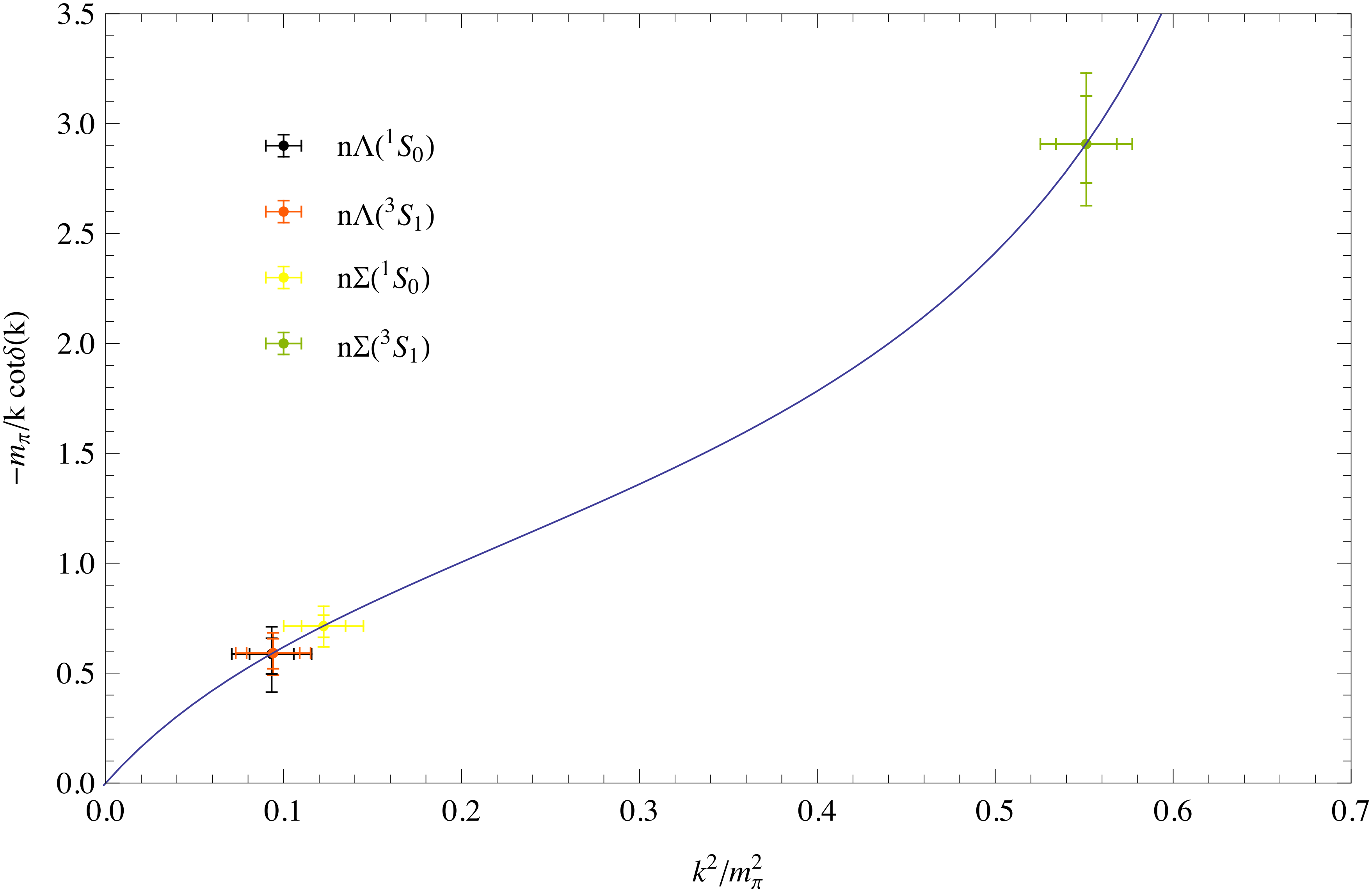
$$\mu_e \simeq 200 \text{ MeV}$$

$$\mu_{\Sigma^-} = m_{\Sigma^-} + \Delta E \lesssim 1290 \text{ MeV}$$

$$\Delta E \lesssim 100 \text{ MeV}$$

Sigma's important (strange quarks)

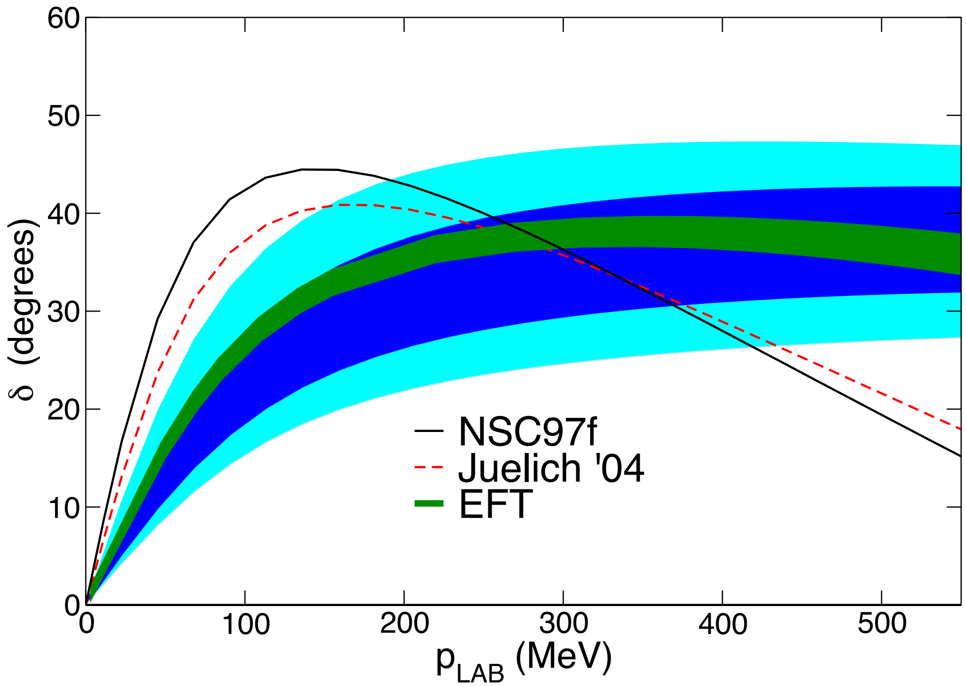
NPLQCD:
PRD 81 (2010)



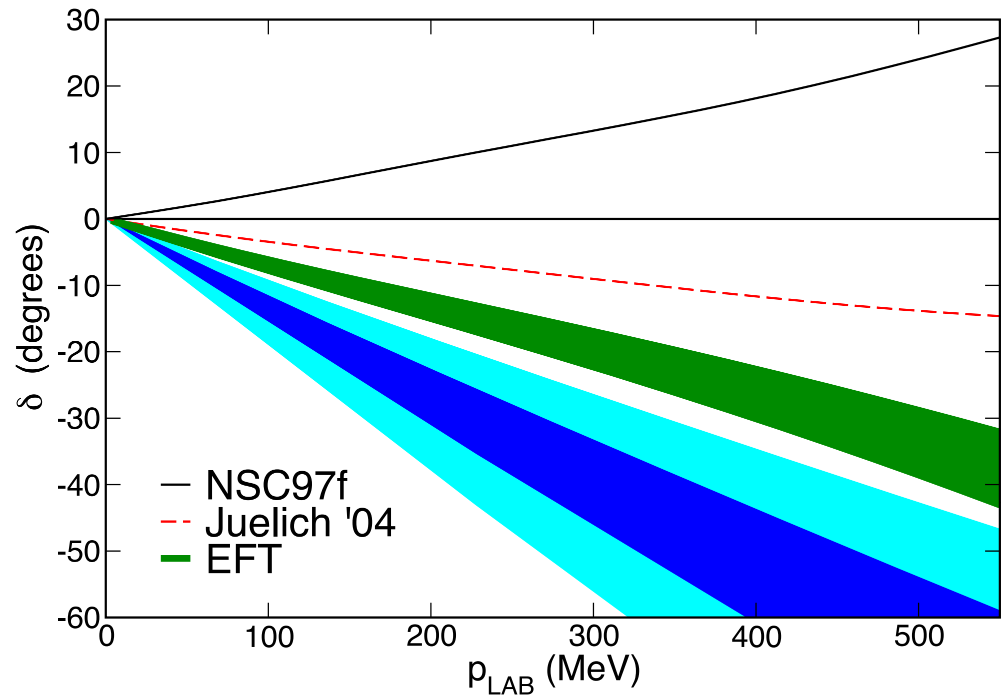
$n\Sigma^-$

NPLQCD: arXiv:1204.3606

Lattice QCD + LO EFT



$^1S_0$

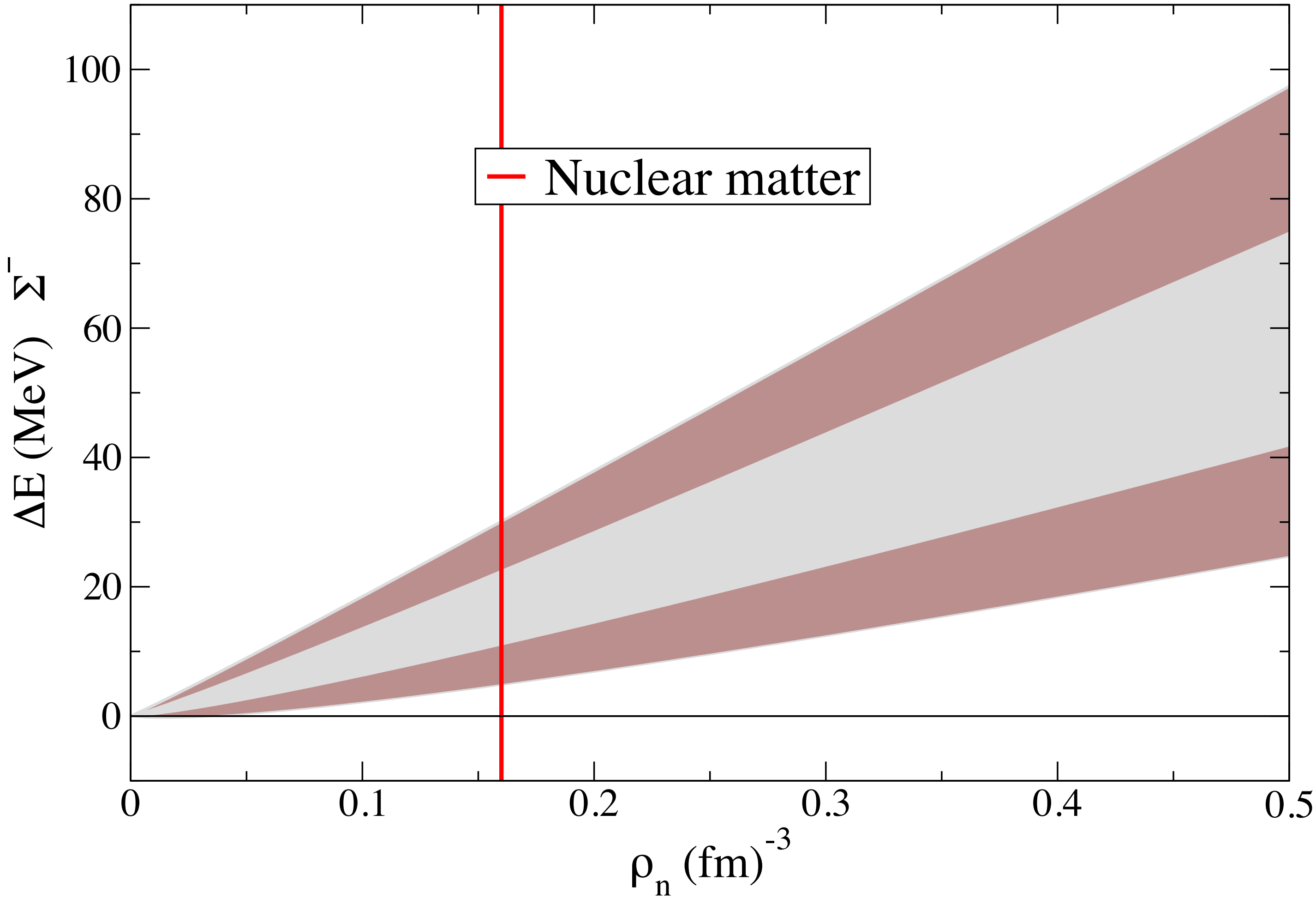


$^3S_1$

$$\Delta E \;=\; -\frac{1}{\pi\mu}\int_0^{k_f} dk\; k\;\left[\;\frac{3}{2}\delta_{^3S_1}(k)\;+\;\frac{1}{2}\delta_{^1S_0}(k)\;\right]$$

$n\Sigma^-$

NPLQCD: arXiv:1204.3606



$$\mu_{\Sigma^-} = m_{\Sigma^-} + \Delta E \lesssim 1290 \text{ MeV}$$

$$\Delta E \lesssim 100 \text{ MeV}$$

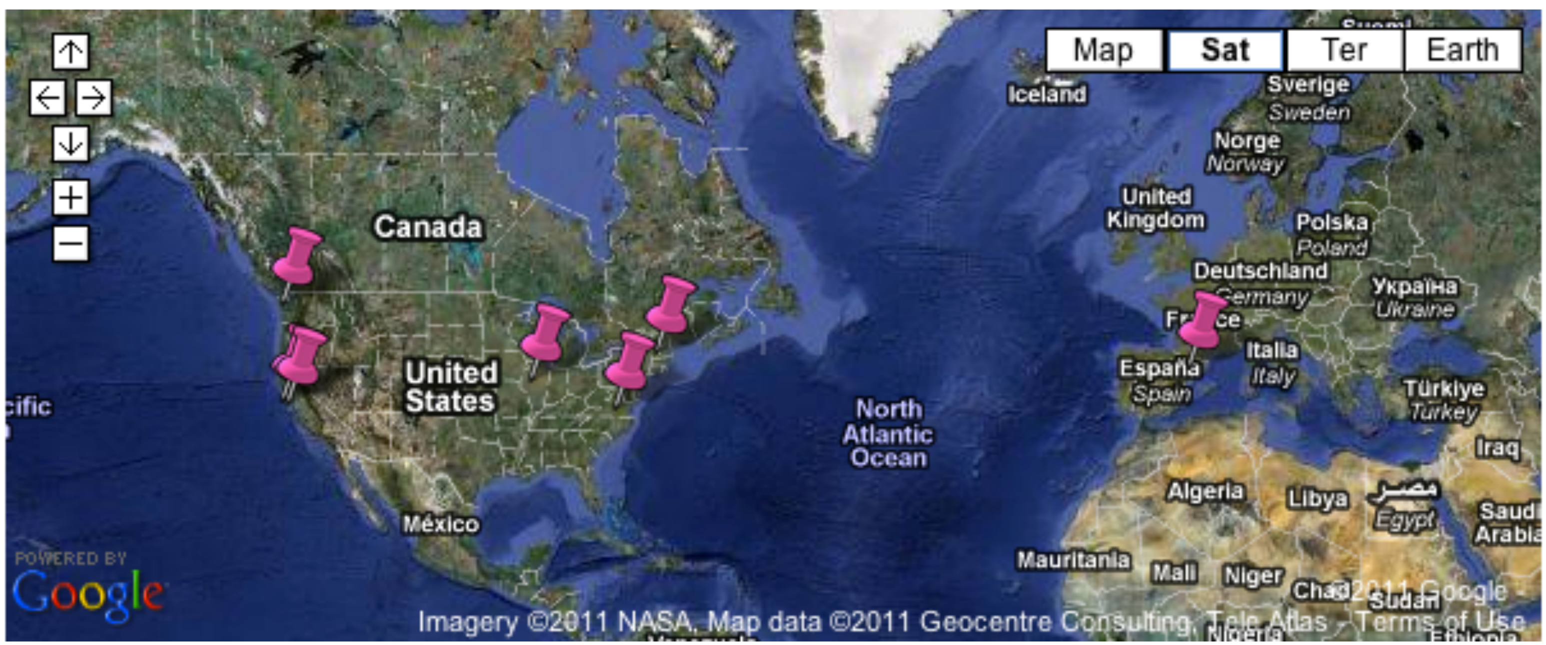
Sigma's important (strange quarks)

Future Outlook

With Lattice QCD we can

- compute $m_n - m_p, B_d,$ etc., as a function of the light quark masses, exploring the observed fine tunings in light nuclei
- compute NNN interactions from QCD
- compute Nucleon-Nucleon, Hyperon-Nucleon and Hyperon-Hyperon interactions from QCD
- in particular, combined with the methods of many-body Effective Theories, we can extend this knowledge to larger nuclei mapping out the quark mass dependence of the Hoyle-state, for example
- nuclear matrix elements: Parity Violating, EDM, ...

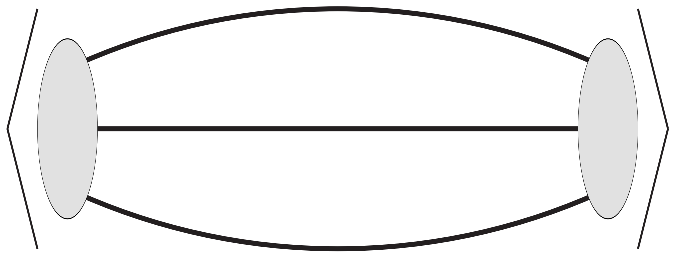
some of my collaborators



Backup

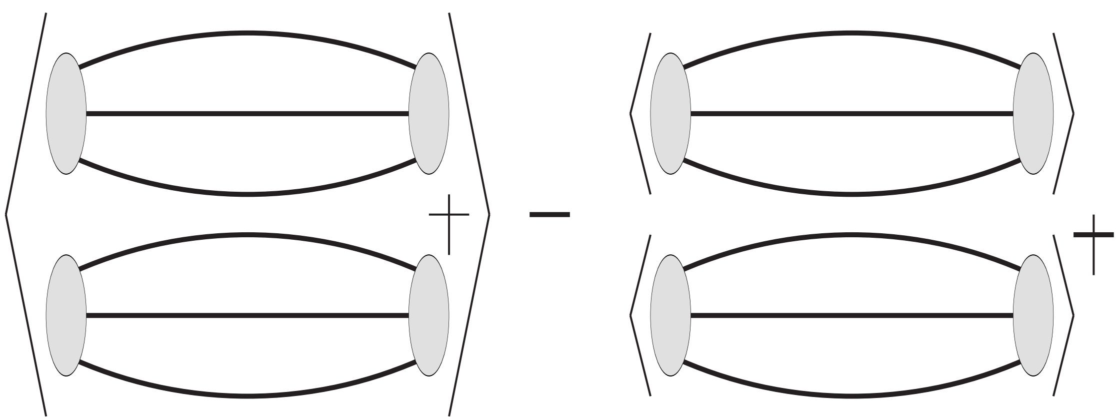
Signal to Noise

$S \sim$



$$\lim_{t\to\infty} S \sim e^{-m_N t}$$

$V \sim$



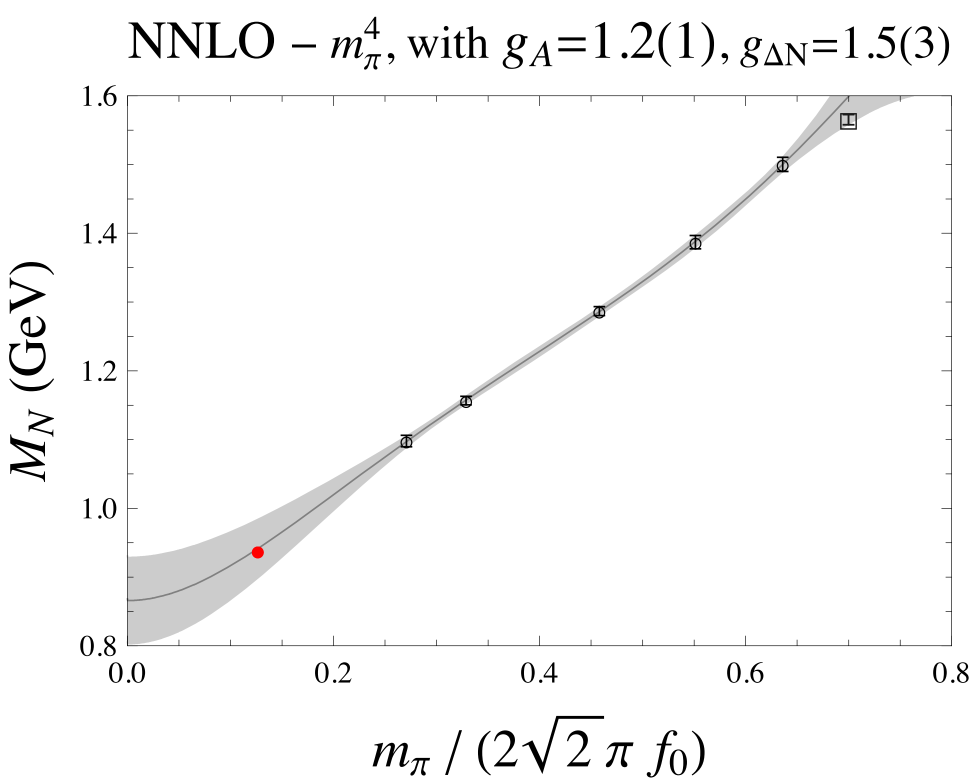
$3\ \pi$ $\qquad$ $NN^\dagger$

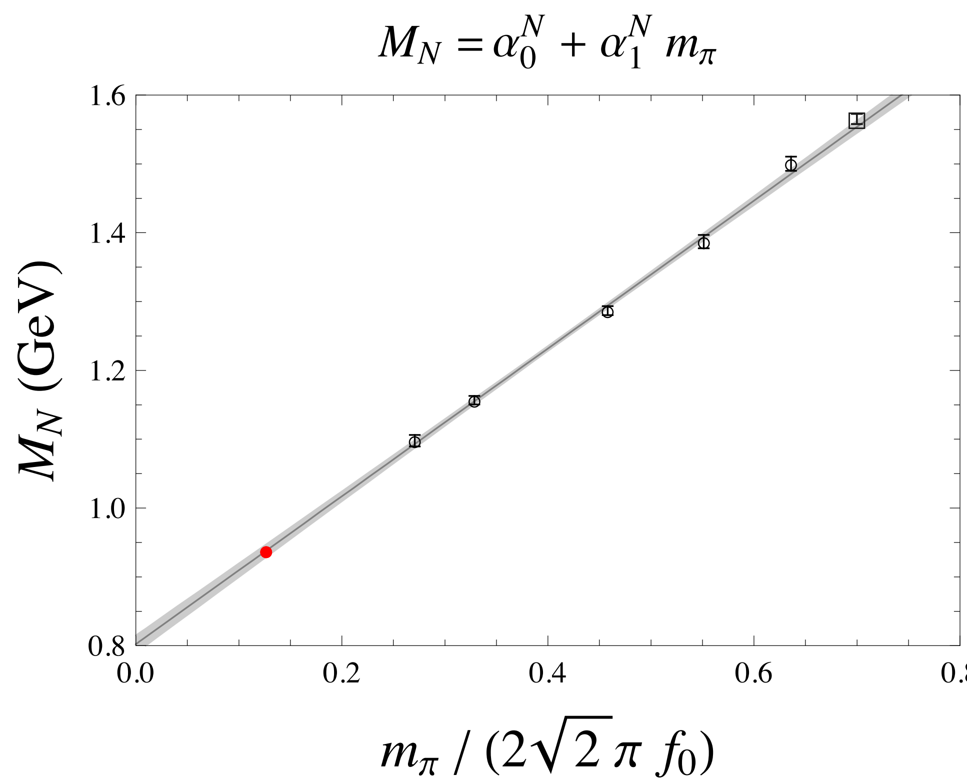
$$\lim_{t\to\infty} \sigma^2 \sim e^{-3m_\pi t}$$

$$\lim_{t\to\infty} \frac{S}{\sigma} \sim e^{-\left(m_N - \frac{3}{2}m_\pi\right)t}$$

Unexpected quark mass dependence of hadronic observables is common in lattice QCD calculations

AWL with LHPC: PRD 79 (2009)





NNLO χPT

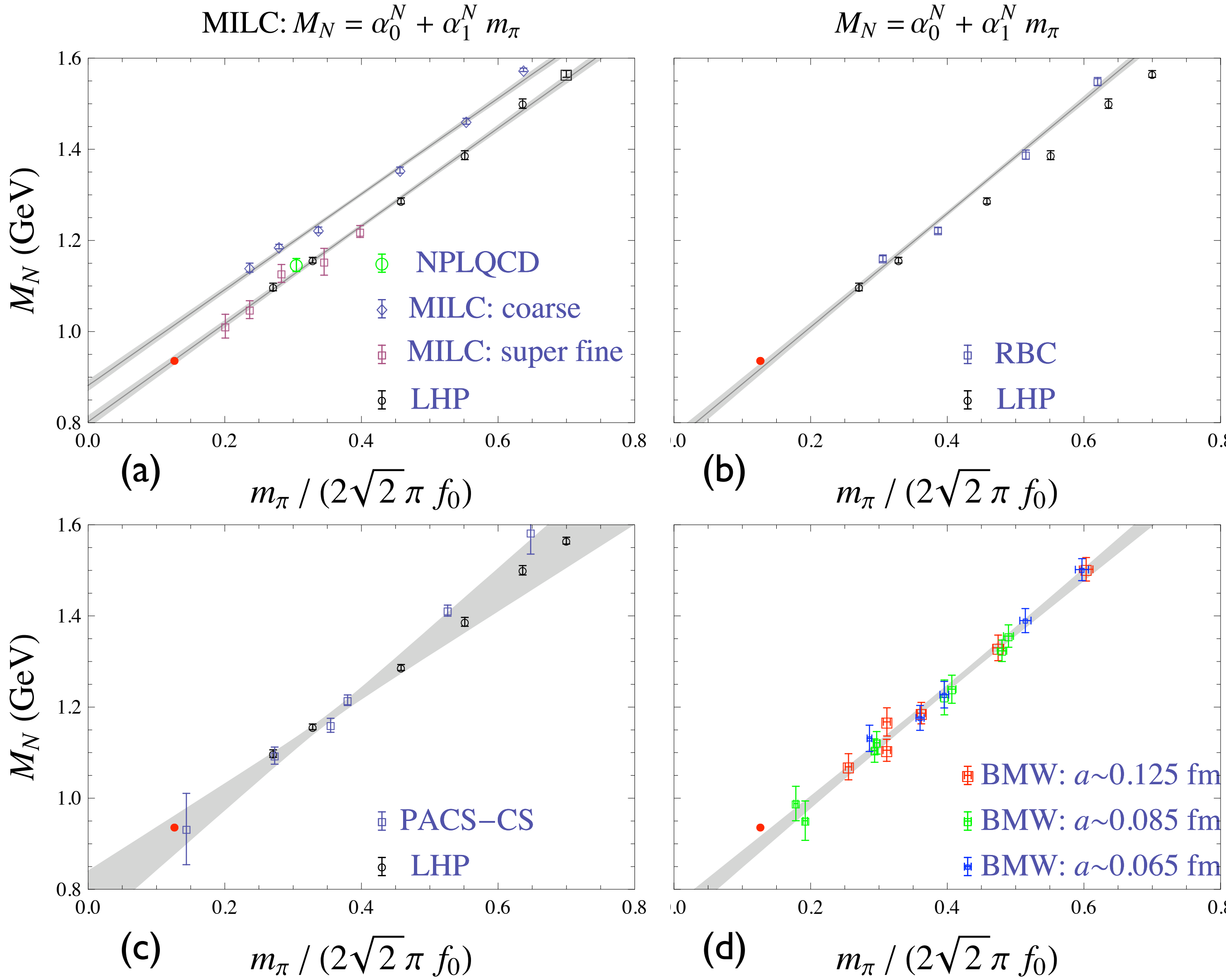
Ruler Approximation!

$$M_N = 941 \pm 42 \pm 17 \text{ MeV}$$

$$M_N = 938 \pm 9 \text{ MeV}$$

Large cancelations between different orders

Linear in pion mass behavior observed in all dynamical lattice calculations with 2+1 flavors



AWL: Lattice 2008 arXiv:0801.0663